MAGYAR TUDOMÁNYOS AKADÉMIA
KÖZGAZDASÁGTUDOMÁNYI INTÉZET

INSTITUTE OF ECONOMICS
HUNGARIAN ACADEMY OF SCIENCES

MŰHELYTANULMÁNYOK **DISCUSSION PAPERS**

MT–**DP.** 2004/19

# THE MORE WE KNOW, THE LESS WE AGREE: PUBLIC ANNOUNCEMENTS AND HIGHER-ORDER EXPECTATIONS

PÉTER KONDOR

Institute of Economics
Hungarian Academy of Sciences

Budapest

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# The more we know, the less we agree: public announcements and higher-order expectations

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Very preliminary, please do not quote!
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KONDOR PÉTER

# MINÉL TÖBBET TUDUNK, ANNÁL KEVÉSBÉ ÉRTÜNK EGYET: BEJELENTÉSEK ÉS MAGASABB-RENDŰ VÁRAKOZÁSOK

## *Összefoglalás*

*Közismert jelenség, hogy a pénzügyi piacokon a bejelentéseket gyakran hektikus kereskedelem, magas forgalom és volatilis árak kísérik. Ezt általában a bejelentést követő növekvő bizonytalanság jeleként értelmezik. Ugyanakkor az irodalomban elterjedt az az érvelés, hogy a bayesi tanulással nem összeegyeztethető, hogy egy mindenki által megfigyelhető bejelentés növelje a bizonytalanságot. Ebben a dolgozatban bemutatjuk, hogy ez a jelenség nem csak hogy bizonyos információs helyzetekben konzisztens a bayesi modellel, de – ha feltesszük, hogy a piaci szereplők a jövőben el akarjak adni a most megvásárolt értékpapírokat, tehát megpróbálják eltalálni a jövőben kereskedő szereplők értékelését – vannak olyan információs helyzetek, amelyek egyszerűek, természetesek és maguktól érthető módon eredményeznek bejelentéseket követően növekvő bizonytalanságot még a legegyszerűbb Grossman–Stiglitz modellben is.*

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# THE MORE WE KNOW, THE LESS WE AGREE:
## PUBLIC ANNOUNCEMENT AND HIGHER-ORDER EXPECTATIONS

BY PÉTER KONDOR

***Abstract***

*The stylized fact that public announcements in financial markets are followed by intense trading, high trading volume and volatile prices, is widely perceived as the sign of increasing disagreement due to the announcement. However, it is common to argue that this would be inconsistent with Bayesian-learning and common priors. In this paper, we not only show that – with cetan information structures – it is possible in a Bayesian model, but we also argue that with the assumption that traders trade for resale – so they try to second guess future traders' guesses – there are information structures which are simple, intuitive and plausible and result in increasing disargreement even in a standard, multi-period Grossmann–Stiglitz model.*

*Keywords: confirmatory bias, public announcements, trading volume, higher-order expectation, short-term traders*

*JEL classification: D4, D8, G11, G12*

## 1 Introduction

It is a well established stylized fact in financial markets that public announcements are followed by intense trading, high trading volume and volatile prices. It is widely perceived as the sign that the public announcement increases disagreement, and the polarized asset valuations are channeled into prices by the hectic trading activity. (Evans and Lyons, 2003 Love and Payne,2003, Fleming and Remolona,1999, Bamber et al, 1997, Kim and Verecchia 1997, 1994, 1991, Kandel and Pearson, 1995, Harris and Raviv, 1993, Varian, 1989). It is also common to argue that agents' different reaction to the same public information cannot be explained with a standard Bayesian-learning model with common priors. Varian (1989), Harris and Raviv (1993) and Kandel and Pearson (1995) assumes that agents interpret the same information differently, because they have different priors, while Evans and Lyons (2003) and Kim and Verecchia (1997, 1993) suggest a model where the announcement incorporates a private information element i.e. agents look at the same piece of information but they see something different. Relatedly, in behavioral economics similar phenomena[1] are explained by the so-called confirmatory bias: each agent tend to interpret the same information to support his or her own view. Rabin and Schrag (1999) put it as follows:

> "... a large and growing body of psychological research suggests that the way people process information often departs systematically from Bayesian updating. In this paper we formally model and explore the consequences of one particular departure from Bayesian rationality: confirmatory bias. [...] The most striking evidence for the confirmatory bias is a series of experiments demonstrating how providing the same ambiguous information to people who differ in their initial beliefs on some topic can move their beliefs farther apart." Rabin and Schrag(1999, pp 38,43.)

In this paper, we show that with certain information structures a public announcement can increase disagreement even with standard Bayesian decision makers and common priors. More importantly, we argue that with early traders who buy for resale – so who trade on their expectation of the expectation of future traders – there are such information structures for financial markets which are simple, intuitive and plausible. Hence, we present a Grossman–Stiglitz type standard rational expectation model, where public announcements increase disagreement and it generates large trading volume. We also show that our model is consistent with recent evidence from high-frequency data (both FX and Government bond) that although at the time of the announcement there is an initial price adjustment, it is followed by a prolonged period with increased, more volatile and more informative trading (Evans and Lyons, 2003 Love and Payne,2003, Fleming and Remolona, 1999, Love 2004). There is also some direct evidence – using analysts forecast as a proxy for traders opinion – from equity markets (Bamber et al, 1997), which shows– in line with our model – that the increased trading after announcements is indeed associated with increasing disagreement.

The intuition why expectations on expectations of others – higher-order expectations – can be useful to explain increasing disagreement is simple. In the interesting cases of disagreement increases, some agents react contrary to the public announcement: the same information will be good news for one agent and bad news for an other agent. However, it is a quite natural intuitive requirement to assume that good public signals should be associated with strong fundamentals and vice versa, regardless of the private information. The twist in our model, that our early traders will not be interested in the true value, they will be interested in the opinion of future traders about the true value. Hence, high public signals can continue to be associated with strong fundamentals; it is enough if high public signals are associated with a low average private signal of future traders for some early traders and a high one for other early traders. We show that precisely this will be the case, if the

[1] In a typical experiment (conducted by Lord, Ross, and Lepper in 1979) two groups of people were given a sequence of studies on the merits of capital punishment as a deterrent to crime. Individauls in different groups had opposite initial opinion about the issue. After seeing exactly the same information, both groups got even more convinced of their initial opinion.

connection between private information sets of agents in different periods is not too strong, but the public signal is related to all agents private information. In this case the public information will connect early traders' private information to future traders private information. Therefore, before the public announcement, there will not be too much dispersion among differently informed early traders' guesses on late comers average guess, because early traders' private information will not be very informative on late comers' information. However, at the moment of the public announcement, their private information gets connected, so the informational differences among early traders spreads out their asset valuation as well. Hence, it is the existing private information which becomes relevant – in guessing future traders expectation – due to the announcement and channeled into prices by the increased trading activity. Different traders see the same public signal: it just gives a significance to their existing private information, and makes them to trade on it.

Our model fits well to the recent flow of applied theoretical papers analyzing the effect of higher-order expectations on financial markets[2]. The leading metaphor in this literature is the famous beauty contest example of Keynes which compares speculative trading to those beauty contests where gifts are distributed among those who voted for the winner. Similarly to the metaphor with the contestants, the problem of speculative traders is to choose those assets which future traders will consider valuable – so the resale price will be high –, which do not necessarily coincide with those that they consider undervalued. The main observation of these papers is that higher-order expectations in asymmetric information environments may behave very differently to first-order expectations i.e. the low of iterated expectations is violated in a systematic way. This fact in turn, can explain stylized facts of financial markets in a novel way. The first paper in this literature is Allen, Morris and Shin (2003), which shows that assuming short-horizon traders in the standard dynamic asymmetric information model of Brown and Jennings (1989) implies that prices will be oversensitive to public information in the early periods, because higher-order expectations overreacts the public signal. Kondor (2004) – similarly to our model – allows for an information structure, where private information sets of early traders are less connected. As a consequence, expectations on the resale price (higher-order expectations) can move in the opposite to expectations on the fundamental value (first-order expectations) as traders' private information change. Hence, early traders – instead of correcting mispricing – will buy overvalued assets and sell undervalued assets even if future traders are rational. The driving force in our current work is the fact that dispersion of higher-order expectations can increase after an announcement even when the dispersion of first-order expectations decrease.

Our work is naturally related to the theoretical work on trading volume and public information releases. The literature can be divided into two groups. Rational models – models with Bayesian updating and common priors (Brown and Jennings, 1989, Kim and Verecchia, 1991, He and Wang, 1995) – do not deliver disagreement increasing announcements, consequently they have problems explaining the empirical regularities. The basic structure of these models are nested in our set up, so

[2] This literature originates from the application of results from the global games literature on currency crisis by Morris and Shin (1998), but recently it has been extended to non-global games environments. In particular, monetary economics seems to be a fruitful area in the higher-order expectations literature (see Woodford, 2001, Hellwig 2002, Adams, 2003, Amato and Shin 2003).

we will be able to point out the difference which makes our model capable of explaining high trading volume around announcement. Non-Bayesian models in contrast (Varian, 1986, Harris and Raviv, 1993, Kandel and Pearson, 1995, Evans and Lyons, 2001) can produce disagreement and volume, but for the expense of assuming non-common priors or different perception of the same public information. They argue that as rational models are inherently incapable of explaining the observed stylized facts, these assumptions are necessary. We will show that rational models can deliver similar findings with the help of higher order expectations.

The structure of this paper is as follows. In the next section we discuss the characteristic properties of the interesting information structures with increasing disagreement in general, and their relation to trading volume and higher-order expectations. For expositional purposes, in that section we abstract from the informational role of prices and we use a simple dynamic structure. In section 3, we present the full model with learning from prices and more reasonable dynamics and we discuss the results. In section 4 we confront our findings with existing empirical results. Finally we conclude.

## 2 Higher-order expectations and information structures with increasing disagreement

Before discussing the financial applications, let us explore the interesting cases of increasing disagreement due to public information in general. We consider agents $j$ who have to form opinion on an issue. First, let us assume that there are only two agents $j = A, B$. We assume a two step process. They start with the same priors, but in the first step each receives some private information, $I_j$. This private information is responsible for the initial disagreement. We ask them their opinion, $o_j$ at that point. Then a public piece of information, $z$ is also revealed. We ask their opinion, $o'_j$, again. We are interested in the change of agents' opinion due to the public information release. The issue will be represented by the random variable $\phi$.Their initial opinion will be their expectation of $\phi$ given their private information sets:

$$o_j = E(\phi | I_j) \qquad j = A, B.$$

Their final opinion is

$$o'_j = E(\phi | I_j, z) \qquad j = A, B.$$

With two agents, disagreement can increase after public announcements in four different ways. It is possible that the opinion of both agents improve, but the optimist's improve more. The same is possible into the opposite direction. It is also possible that each agent gets even more convinced of his or her original opinion, and finally, disagreement can also increase when optimist becomes the pessimist and vice versa and the change is large enough. The first two cases are rather qualitative than quantitative phenomena, and they occur quite naturally for certain announcements in most information structures, so we will not deal with them in this discussion. We will focus on the more surprising last two possibilities when the public information moves opinions very differently across agents: polarization and belief swap.

**Definition 1** *There is increasing disagreement for $A, B$ and information $z$ if* $|o'_A - o'_B| > |o_A - o_B|$, *there is polarization for $A, B$ and $z$ if $o_A > o_B$ implies*

$$o'_A > o_A \text{ and } o'_B \leq o_B.$$

*There is belief swap if $o_A > o_B$ implies $o'_A < o'_B$.*

It should be clear that polarization implies increasing disagreement, and belief swap can happen with and without increasing disagreement. In our full model the public signal will cause belief swaps together with increasing disagreement for any two agents with different private signals and for any announcement.

To illustrate our concepts, we present the following example. There are two possible public announcements and one of them causes belief swap while the other causes polarization and both of them results in increasing disagreement.

**Example 1** [3]*Suppose that investors are waiting for the opinion of a financial analyst on a particular firm. The firm can be a valuable one (V) or a worthless (W) one . However, the credit rating agency also can have two types. It is either an enemy (E), who always gives bad advice, – possibly because some conflicts of interest as a consequence of being a branch of an investment bank – or a friend (F) who always gives good advice. The prior distribution about these states is the same for all investors:*

$$\begin{array}{ccc} & V & W \\ F & p & \frac{1-2p}{2} \\ E & \frac{1-2p}{2} & p \end{array}$$

*where $\frac{1}{4} < p < \frac{1}{2}$. So the a priori chance of the firm being valuable is $\frac{1}{2}$, but – for some reason – there is some correlation between the type of the analyst and the type of the firm: valuable firms tend to go to friend credit agencies and vice-versa. There are two steps of information arrivals. First each investor receives a noisy information $i$ about the type of the analyst. Hence, it is either $i_j = F$ or $i_j = E$. This signal is true with probability $1 > q > \frac{1}{2}$. After this signal, we ask $A$ and $B$ of their probability assessment of the firm being a valuable investment possibility: $o_j = \Pr(V|i_j)$. In the second step the analyst announces its report which is either that the firm is valuable $z = V$ or that the firm is worthless $z = W$. We assume that the agency knows the value of the firm for certain. Hence, the opinion over the firm after the announcement will be defined as $o'_j = \Pr(V|i, z)$ Let us assume that investor $A$ received the information $i_A = F$, while investor $B$ received the private information $i_B = E$. Hence,*

$$\begin{aligned} o_A &= \frac{qp + (1-q)\left(\frac{1-2p}{2}\right)}{qp + (1-q)\left(\frac{1-2p}{2}\right) + q\frac{1-2p}{2} + (1-q)p} = 4pq - q - 2p + 1 \\ o_B &= \frac{(1-q)p + q\left(\frac{1-2p}{2}\right)}{(1-q)p + q\left(\frac{1-2p}{2}\right) + (1-q)\frac{1-2p}{2} + qp} = 2p + q - 4pq \end{aligned}$$

[3] *We thank Enrico Sette for this interpretation of our example.*

*,so*

$$o_A - o_B = (4p-1)(2q-1) > 0$$

*Now, assume that the announcement of the analyst is that the firm is good: $z = V$. Then*

$$\begin{aligned} o'_A &= \frac{qp}{qp + (1-q)p} = q \\ o'_B &= \frac{(1-q)p}{qp + (1-q)p} = 1-q \end{aligned}$$

*hence*

$$\begin{aligned} o'_A - o_A &= (1-2p)(2q-1) > 0 \\ o'_B - o_B &= -(1-2p)(2q-1) < 0. \end{aligned}$$

*Therefore, we have polarization.*

*Now, let us assume that the analyst announces that the firm is worthless: $z = W$.Then*

$$\begin{aligned} o'_A &= \frac{(1-q)\frac{1-2p}{2}}{q\frac{1-2p}{2} + (1-q)\frac{1-2p}{2}} = 1-q \\ o'_B &= \frac{q\frac{1-2p}{2}}{q\frac{1-2p}{2} + (1-q)\frac{1-2p}{2}} = q \end{aligned}$$

*so $o'_B > o'_A$ and there is belief swap.*

*Furthermore, whichever is the public signal*

$$\left|o'_A - o'_B\right| - |o_A - o_B| = (q-(1-q)) - ((4p-1)(2q-1)) = 2(1-2p)(2q-1) > 0$$

*Hence, there is increasing disagreement for any announcement.*

There are two critical points in this example, which resulted in increasing disagreement and which will be present in our model as well. The first one is that – conditionally on the pay-off relevant state – the public information is not independent of the private information : the announcement of the analyst means something different for the two agents. The second one is that the public information made the private information much more relevant: knowing the type of the analyst does not help much if she does not announce her rating.

From now on, let us assume that the private piece of information of each individual $i$, consists of a single private signal $x_i$, and there are infinitely many types. The distribution function of the pay-off relevant state $\phi$, the vector of public signals $z$ and the private signals $x_i$ is distributed by the density function $f(\phi, x_i, z)$. Hence, for the sake of simplicity, we assume that conditionally on $\phi$ and $z$, the private signals are drawn independently from the same distribution. Furthermore, we assume that the marginal density function of any subset of our random variables exists and also differentiable with respect to any of our variables and all of these densities have a full support on a given closed state space $S \subseteq R^{2+n_z}$ where $n_z$ is the dimension of $z$ [4]. Let us also assume that $\frac{\partial^2 \ln f(\phi|x_i)}{\partial\phi\partial x_i}$ so from Milgrom (1981)[5] $E(\phi|x_i)$ is increasing in $x_i$. It is then not too hard to give simple sufficient conditions for belief swap and polarization.

**Proposition 1** *1. If*

$$\begin{aligned}\frac{\partial \ln f(z|\phi, x'')}{\partial\phi} &> 0 \text{ but} \\ \frac{\partial \ln f(z|\phi, x')}{\partial\phi} &< 0 \text{ for all } \phi\end{aligned}$$

*then there is polarization for the given $x'' > x'$ and $z$., where $x', x'', z, \phi \in S$.*

*2. If*

$$-\frac{\partial^2 \ln f(z|\phi, x)}{\partial\phi\partial x} < \frac{\partial^2 \ln f(\phi|x)}{\partial\phi\partial x}$$

*for the given $z$ and all $\phi, x \in S$ then there is belief swap for $z$ and all $x$.*

**Proof.** From Milgrom (1981), if $\frac{\partial \ln g_1(\phi)}{\partial\phi} > \frac{\partial \ln g_2(\phi)}{\partial\phi}$ where $g_1(\cdot), g_2(\cdot)$ are two distribution functions, then $E_1(\phi) > E_2(\phi)$ holds. Hence, for polarization, it is sufficient if

$$\frac{\partial \ln f(\phi|x'', z)}{\partial\phi} > \frac{\partial \ln f(\phi|x'')}{\partial\phi} \text{ and } \frac{\partial \ln f(\phi|x', z)}{\partial\phi} < \frac{\partial \ln f(\phi|x')}{\partial\phi}.$$

Similarly, for belief swap it is sufficient if

$$\frac{\partial^2 \ln f(\phi|x, z)}{\partial\phi\partial x} < 0 < \frac{\partial^2 \ln f(\phi|x)}{\partial\phi\partial x}.$$

But

$$\begin{aligned}\ln f(\phi|x, z) &= \ln\frac{f(\phi, x, z)}{f(x, z)} = \ln\frac{f(z|\phi, x) f(\phi|x)}{f(z|x)} = \\ &= \ln f(z|\phi, x) + \ln f(\phi|x) - \ln f(z|x),\end{aligned}$$

which – by substituting back to the inequalities above – gives all the results. ■

Our conditions are in line with the intuition provided by our example. There is polarization if the public announcement is good news if the agent knows $x''$ but bad news if she knows $x'$.[6] Furthermore,

[4] The generalization of any of the results below for the discrete case would be a straitforward excercise.

[5] It is that $f(\theta|x_i)$ is log-supermodular in $\theta$ and $x_i$ or that the loglikelihood ratio property is satified: $\frac{f(\theta''|x'')}{f(\theta'|x'')} > \frac{f(\theta''|x')}{f(\theta'|x')}$ for all $\theta'' > \theta'$ and $x'' > x'$. As it is shown in Milgrom (1981), this is sufficient condition for second order stochastic dominance and consequently sufficient for $E(\theta|x)$ to be increasing in $x$.

[6] Milgrom (1981) uses the definition that news $z''$ is more favourable than $z$ iff

$$\begin{aligned}\frac{f(z''|\theta'')}{f(z''|\theta')} &> \frac{f(z'|\theta'')}{f(z'|\theta')} \text{ or} \\ \frac{\partial \ln f(z''|\theta)}{\partial\theta} &> \frac{\partial \ln f(z'|\theta)}{\partial\theta}.\end{aligned}$$

there is belief swap if the larger $x$, the worse news is $z$ and if this effect is strong relative to the effect of the private signal on the probability of $\phi$ in the absence of public information.

Unfortunately, it is a bit harder to find simple conditions for increasing disagreement in general and increased trading volume due to increasing disagreement in particular, so we add some structure to our set up. So let us turn to our financial application and assume that our agents are traders in a market of a risky asset. Instead of the effect of the public announcement on their opinion, we will interested in its effect on their demand for the risky asset. Each agent has a constant absolute risk-aversion utility $U_j(W_j) = -e^{-a_j W_j}$, where $W_j$ is their wealth when they exit, and $a_j$ is the measure of risk-aversion. Here, the "issue", $\phi$, which they form opinion on, will be the value of the asset when they leave the market. Hence, $\phi$ can differ across traders. In particular, we will assume that each trader belongs to one of the $t = 1...T$ groups and these groups arrive to the market sequentially. There is a continuum of traders in each group. Traders in the same group arrive to the market in the same time, observe the announcement, $y$, if there is one, trade with the other members of the group, and finally, they sell all of their position to the next group which has just arrived to the market. Hence, a trader $j$ in group $t$ will be interested in the equilibrium price of the trading session of the next group: $\phi_{jt} = \phi_t = p_{t+1}$. The true value of the asset, $\theta$, will be realized only after the last group has traded, hence only last period traders are interested directly in the true value i.e. $\phi_{jT} = \phi_T = \theta$. Each agent submits a demand function based on her private signal, $x_{jt}$ past and present equilibrium prices, $p_t, p_{t-1}, ...$, and the public announcement, $y$, if there was one. We are interested in the difference between the aggregate demand of traders with and without announcement. In each trading session, there is a random supply of the asset and it is independent across sessions, so prices are never fully revealing. We assume that all random variables are jointly normal and that traders in the same group are symmetric i.e. the distribution of $x_{it}$ conditional on the other variables are the same across all $i$ in period $t$.

Then the demand of trader $j$ in period $t$ will be

$$d_{jt} = \frac{E\left(\phi_t | x_{jt}, p^t\right) - p_t}{a_{jt} var\left(\phi_{jt} | x_{jt}, p^t\right)}$$

if there is no announcement and

$$d'_{jt} = \frac{E\left(\phi_t | x_{jt}, p'^t, y_t\right) - p'_t}{a_{jt} var\left(\phi_t | x_{jt}, p^t, y_t, p'_t\right)}$$

if there is announcement, where $p^t$ is the price history up to $t$ and $p'_t$ is the equilibrium price with announcement. Intuitively, it is clear that the speculative trading will be larger[7], if each trader trades

In that sense, our sufficient condition for polarization can be interpreted that $z$ is more favourable when our agent knows $x''$ than if she knows $x'$.

[7] We will be more precise on the diffinition of trading volume and its relation to speculative trades and risk-sharing trades later in the text. The present discussion intend to remain on the intuitive level.

more aggressively due to the announcement i.e.

$$\left|\frac{\partial d_{jt}}{\partial x_{jt}}\right| < \left|\frac{\partial d'_{jt}}{\partial x_{jt}}\right|.$$

Because of the linearity of demand functions, this will be satisfied either for all agents or for none of them. Furthermore, – because the variance is independent of the realized signal – this is exactly the condition for increasing disagreement between any two of the agents in the same group.

For the rest of the section, let us assume that traders do not learn from past and present prices. There will be learning from prices in the full model, here we abstract it away only to strenghten our intuition. Now, we can make two observations. The first one is, that with normally distributed variables

$$\begin{aligned}
\frac{\partial d_{jt}}{\partial x_{jt}} &= \frac{1}{a_{jt}} \frac{\frac{\partial E(\phi_t|x_{jt})}{\partial x_t}}{var\left(\phi_t|x_{jt}\right)} = \frac{1}{a_{jt}} \frac{\partial^2 \ln f\left(\phi_t|x_{jt}\right)}{\partial \phi_t \partial \theta} \\
\frac{\partial d'_{jt}}{\partial x_{jt}} &= \frac{1}{a_{jt}} \frac{\frac{\partial E(\phi_t|x_{jt},y)}{\partial x_t}}{var\left(\phi_t|x_{jt},y\right)} = \frac{1}{a_{jt}} \frac{\partial^2 \ln f\left(\phi_t|x_{jt},y\right)}{\partial \phi_t \partial \theta}
\end{aligned}$$

As we know that

$$\ln f\left(\phi_t|x_{jt},y\right) = \ln f\left(y|\phi_t,x_{jt}\right) + \ln f\left(\phi_t|x_{jt},y\right) - \ln f\left(y|x_{it}\right),$$

we also know that

$$\frac{\partial d'_{jt}}{\partial x_{jt}} = \frac{\partial d_{jt}}{\partial x_{jt}} + \frac{1}{a_{jt}} \frac{\partial^2 \ln f\left(y|\phi_t,x_{jt}\right)}{\partial \phi_t \partial x_{jt}} = \frac{\partial d_{jt}}{\partial x_{jt}} - \frac{\frac{\partial E(y|\phi_t,x_{jt})}{\partial x_t} \frac{\partial E(y|\phi_t,x_{jt})}{\partial \phi_t}}{a_{jt} var\left(y|\phi_t,x_{jt}\right)}.$$

With the logic of proposition 1, this equation gives necessary and sufficient conditions for increasing disagreement with belief swap in the following proposition.

**Proposition 2** *When prices are not informative, traders will reverse their bets due to the announcement, if and only if*

$$\frac{\frac{\partial E(y|\phi_t,x_{jt})}{\partial x_t} \frac{\partial E(y|\phi_t,x_{jt})}{\partial \phi_t}}{a_{jt} var\left(y|\phi_t,x_{jt}\right)} > \frac{\partial d_{jt}}{\partial x_{jt}}$$

*and they also trade more aggressively if and only if*

$$\frac{\frac{\partial E(y|\phi_t,x_{jt})}{\partial x_t} \frac{\partial E(y|\phi_t,x_{jt})}{\partial \phi_t}}{a_{jt} var\left(y|\phi_t,x_{jt}\right)} > 2\frac{\partial d_{jt}}{\partial x_{jt}}.$$

The last expression shows, why our structure, with early-traders and late-comers can result in increasing trading activity after announcement as opposed to other models. In our last period, $\phi_T = \theta$. This is the period which coincides to a model without short-term traders. Now, with the usual assumption that the announcement is a noisy version of the true value, $y = \theta + \eta$, the private signal cannot possibly improve the estimation of the public signal beyond the true value, i.e. $\frac{\partial E\left(y|\phi_T,x_{jT}\right)}{\partial x_T} = 0$.

So our condition cannot hold. But in period $T-1$, trader are interested in the last period price, not the true value: $\phi_{T-1} = p_T$.which will be a function of the information of last period traders only. Is it possible that $x_{jT-1}$ gives additional information on $y = \theta + \eta$ apart from the information contained in $p_T$? Yes, if the private signal of traders in period $T-1$gives information of a different aspect of $\theta$ which is not contained in the information set of last period traders.

Let us approach our problem from a final direction. It is clear that in each trading session $t$, the price will be close to the average opinion about $\phi_t$ (see Allen et al, 2003) i.e.

$$\begin{aligned} \phi_{T-1} &= p_T \approx \bar{E}(\theta) \\ \phi_{T-2} &= p_{T-1} \approx \bar{E}(p_T) \approx \bar{E}\left(\bar{E}(\theta)\right) = \bar{E}^2(\theta) \\ \phi_{T-k} &= p_{T-k+1} \approx \bar{E}^k(\theta) \end{aligned}$$

Hence, traders in period $T-k$ will trade more aggressively after getting the public information if

$$\left|\frac{\partial d_{jT-k}}{\partial x_{jT-k}}\right| = \left|\frac{1}{a_{jt}} \frac{\frac{\partial E\left(\bar{E}^k(\theta)|x_{jt}\right)}{\partial x_t}}{var\left(\phi_t|x_{jt}\right)}\right| < \left|\frac{1}{a_{jt}} \frac{\frac{\partial E\left(\bar{E}^k(\theta)|x_{jt},y\right)}{\partial x_t}}{var\left(\phi_t|x_{jt},y\right)}\right| = \left|\frac{\partial d'_{jT-k}}{\partial x_{jT-k}}\right|.$$

As we know that the conditional variance will drop if there is more information, it is enough if the higher-order expectation term gets more sensitive to the private signal. Our last proposition in this section gives the conditions for this. Let us assume the covariances between any two of our random variables are positive. For the sake of simplicity, we will assume a symmetric structure , where the variance of every private signal is $\sigma_x^2$, the covariance between any private signal and the fundamental value or the public signal are $\sigma_{\theta,x}$ and $\sigma_{y,x}$ respectively. Furthermore, the covariance between two private signals in the same period is $\sigma_{x,x}$, while the covariance between private signals in different periods is $\sigma_{x,x'}$. The only assumption we make on the relative sizes of covariance and variance terms is that the private signal $x_t^i$ and the public signal, $y$, are a positive signals[8] on $\theta$ for agent $i$ in period $t$ in the presence of the private signal i.e. a higher $x_t^i$ or a higher $y$ will rise the fundamental expectation of agent $i$:

$$\begin{aligned} b'_\theta &= \frac{\partial E\left(\theta|x_t^i, y\right)}{\partial x_t^i} = \frac{\sigma_{\theta,.x}\sigma_y^2 - \sigma_{x,y}\sigma_{\theta,y}}{\sigma_x^2\sigma_y^2 - \sigma_{x,y}^2} > 0 \\ c'_\theta &= \frac{\partial E\left(\theta|x_t^i, y\right)}{\partial y} = \frac{-\sigma_{x,\theta}\sigma_{x,y} + \sigma_{\theta,y}\sigma_x^2}{\sigma_x^2\sigma_y^2 - \sigma_{x,y}^2} > 0. \end{aligned}$$

It is easy to check that $b_\theta, c_\theta > 0$ implies that the private signal is a positive signal on $\theta$ in the no-announcement case as well – i.e. $b_\theta = \frac{\partial E\left(\theta|x_t^i\right)}{\partial x_t^i} = \frac{\sigma_{\theta,x}}{\sigma_x^2} > 0$ – and that $|b_\theta| > |b'_\theta|$. This condition implies that public announcements will never increase disagreement over the fundamental value i.e. $b'_\theta, c'_\theta > 0$ implies $b_\theta > b'_\theta$. However, the next proposition shows that public announcements will increase disagreement over all higher-order expectations, if private information sets across periods are

[8] In a more general set-up, this assumption would correspond to the assumumption of positive affiliation of $\left(\theta, x_t^i, y\right)$ for all $t$ and $i$. (Our assumption is weaker, but works for the normal case only.)

not too strongly correlated. The intuition behind this result is that with higher order expectations agents' guess on the fundamental value does not matter; the critical point is what each agent knows about another agent's signal.

**Proposition 3** *If $x_t^i$ and $y$ are positive signals i.e. $b_\theta, c_\theta > 0$ and*

$$\frac{b_\theta \rho_{x,y}^2}{\left(b_\theta + \rho_{\theta,x}\left(1-\rho_{x,y}^2\right)\right)} > \rho_{x',x}, \tag{1}$$

*where $\rho_{v_1,v_2}$ is the correlation between $v_1$ and $v_2$ then*

$$\left|\frac{\partial E\left(\bar{E}^k\left(\theta\right)|x_{jt}\right)}{\partial x_t}\right| < \left|\frac{\partial E\left(\bar{E}^k\left(\theta\right)|x_{jt},y\right)}{\partial x_t}\right| \tag{2}$$

*holds for all $k > 0$.*

**Proof.** Let us introduce $b_x'$, and $b_x$ for the coefficients of private signals in traders' conditional expectations on other traders private signals in other periods:

$$\begin{aligned} b_x' &= \frac{\sigma_{x,x'}\sigma_y^2 - \sigma_{x,y}^2}{\sigma_x^2\sigma_y^2 - \sigma_{x,y}^2} = \frac{\partial E\left(x_u^j|x_t^i,y\right)}{\partial x_t^i} \text{ for all } u \neq t, \\ b_x &= \frac{\sigma_{x,x'}}{\sigma_x^2} = \frac{\partial E\left(x_u^j|x_t^i\right)}{\partial x_t^i} \text{ for all } u \neq t. \end{aligned}$$

Note that

$$\left|\frac{\partial E\left(\bar{E}^k\left(\theta\right)|x_{jt}\right)}{\partial x_t}\right| = \left|b_x^k b_\theta\right| \text{ and } \left|\frac{\partial E\left(\bar{E}^k\left(\theta\right)|x_{jt},y\right)}{\partial x_t}\right| = \left|b_x'^k b_\theta'\right|$$

As $b_\theta > b_\theta' > 0$, $|b_x|\, b_\theta < |b_x'|\, b_\theta'$ implies $|b_x| < |b_x'|$. Therefore, $|b_x|\, b_\theta < |b_x'|\, b_\theta'$ implies (2). The condition in the proposition comes directly – after substitution and straightforward manipulation – from $|b_x^n|\, b_\theta^n < |b_x|\, b_\theta$. ■

In this section we showed that increasing disagreement is consistent with common priors and Bayesian decision making. We argued that two types of increasing disagreement are particularly surprising: the one with polarization and the one with belief swap. We concentrated more on the second type, because this one occurs in our model. We showed that a group of sufficient conditions for belief swap for general distributions can be used to derive necessary and sufficient conditions for more aggressive and reverse trading after announcements in Cara-Normal environments without learning from prices. We also showed that these conditions are satisfied when traders trade for reselling their assets if early traders' information is weakly connected to latecomers information.

In the next section we show that the intuition presented in this section goes through in a standard dynamic Grossman-Stiglitz model where all traders learn by Bayesian-updating.

# 3 The model

## 3.1 The set-up

We modify a standard, dynamic, CARA-Normal, rational expectations model with asymmetric information (He and Wang, 1995, Brown and Jennings, 1989, etc.). As in any similar model since Grossman-Stiglitz (1980), preferences of our traders are given by $U_i(W_i) = -e^{-aW_i}$, where $W_i$ is monetary wealth at the time of the exit, $a$ is the absolute risk-aversion parameter and in each period traders submit demand curves to an auctioneer to buy up the random supply of assets: $u_t \tilde{} N\left(0, \frac{1}{\delta_t^2}\right)$. Traders base their portfolio decision on the private signal which they receive at the moment of their entry and all available public signals i.e. past and present prices and public announcements. They update their beliefs by Bayes' Rule. Prices, $p_t$, are determined by market clearing.

However, as a non-standard assumption, we will have two groups of traders – with continuum traders in each group – and 2+1 periods ($t = 0, 1, 2$). Traders in the first group trade among themselves in periods 0 and 1 and sell all of their remaining assets in period 2. Traders in the second group trade among each other in period 2 and liquidate for the uncertain value of $\theta$ at the end of the game. For expositional purposes only, let us interpret our model in terms of a 24-hour day in the USD/GBP market. However, the reader should keep in mind that our set up would fit to any market, where traders focus on the resale value of their assets instead of their fundamental value. With the FX interpretation, the first group represents traders based in London, while the second group is based in New York. Period 0 and 1 are daylight periods in London, so Londoners trade among themselves twice, then they go to sleep, so they sell all their holdings to New Yorkers. They do not hold positions overnight[9]. Period 2 is daylight in New York, so New Yorkers trade among themselves and get $\theta$ in the evening. We assume that if there is a public announcement,$y$, then it will be released at the beginning of period 1. Hence, we will focus on the differences in trading patterns of Londoners with and without announcement.

The driving force of our model lies in the information structure. We assume that the fundamental value of the asset – the exchange rate in this interpretation– is given by

$$\theta = \theta_s + \theta_k + \theta_w$$

where $\theta_s, \theta_k, \theta_w$ are the US factor, the UK factor and the world factor respectively. We assume that the private signal that Londoners receive contains noisy information on the UK factor and the world factor, but does not contain information on the US factor: $x_i = \theta_k + \theta_w + \varepsilon_i$, while the private signals of New Yorkers contain information on the US factor and the world factor, but not on the UK factor: $z_j = \theta_s + \theta_w + \varepsilon_j$. Hence, the world factor simply represents the common element in the information set of agents in different groups, while the US factor and the UK factor represent group-specific information. The public signal contains information on fundamental value: $y = \theta + \eta$.

[9] Although we use the interpretation of a 24-hour FX market only for expositional reasons, it happens to be a stylized fact among FX dealers that they do not hold positions overnight (see Lyons, 2001).

We assume that all factors and noise terms are i.i.d. and normally distributed:

$$\theta_k, \theta_s \sim N\left(0, \frac{1}{\kappa}\right), \theta_w \sim N\left(0, \frac{1}{\omega}\right), \varepsilon_i, \varepsilon_j \sim N\left(0, \frac{1}{\alpha}\right), \eta \sim N\left(0, \frac{1}{\beta}\right).$$

It is instructive to compare our structure to information in other rational expectations models. The following table presents the structure of some of the most prominent models in the literature.

| model | private s. | public s. | liquidation v. |
|---|---|---|---|
| Brown-Jennings(1989) Kim-Verrecchia(1991,1994) | $\theta + \varepsilon_i$ | $\theta + \eta$ | $\theta$ |
| He and Wang(1995) | $\theta + \varepsilon_i$ | $\theta + \eta$ | $\theta + \xi$ |

Note, that all these models – together with the majority of asymmetric information models in finance [10] – use a one-factor framework. The problem is that the assumption that all signals are noisy versions of the fundamental, imposes a very rigid structure on the information sets. Namely, all covariances between any two of the random variables equals the variance of the fundamental value: $cov\,(x_i, x_j) = cov\,(x_i, y) = cov\,(x_i, \theta) = var\,(\theta)$. Our structure represents a partial relaxation of this assumption. We allow for weaker correlation between private signals across groups. We presented the structure with the help of the specific story about the FX traders just for exponential purposes. We believe that our model applies to any financial markets, where traders cannot be sure to be able to hold their positions until it is optimal and where those whom they will sell to, do not necessarily have a very similar information set to their own.

In the one-factor structure disagreement never increases due to the announcement. In particular, in the simplest Brown and Jennings model, there is no effect of public announcement at all: the increased precision and the decreased disagreement exactly cancels out. In the Kim-Verrecchia case there is some volume due to different precision of signals of different traders, but the effect is small and always proportional to the price change. In He and Wang, there is an additional random factor in the liquidation value, $\xi$, which is not included in the union of traders' information set. This induce traders to follow a more complex dynamic strategy over time, which allows traders to bet in advance on the price effect of the public announcement. Hence, they will build up positions before the announcement and liquidate these positions when the announcement is released. This effect works only with expected announcements. Our model will deliver this bet-in-advance effect as well, but we will have an additional effect coming from the increased disagreement.

Our model nests these information structures. The next table summarizes these connections.

| | *priv* | *pub* | *fund* | $p_2(\cdot)$ | model |
|---|---|---|---|---|---|
| $\kappa, \delta_2 \to \infty$ | $\theta_w + \varepsilon_i$ | $\theta_w + \eta$ | $\theta_w$ | $\approx \theta_w$ | two period -one factor, B-J |
| $\kappa \to \infty$ | $\theta_w + \varepsilon_i$ | $\theta_w + \eta$ | $\theta_w$ | $\approx \theta_w + u_2$ | He and Wang (1995) |
| $\omega \to \infty$ | $\theta_k + \varepsilon_i$ | $\theta_s + \eta$ | $\theta_k + \theta_s$ | $\approx \theta_s + u_2$ | independent information sets |

When $\kappa \to \infty$, the non-common factors, $\theta_s, \theta_k$ lose their importance and we end up in a one-factor structure with $\theta_w$ only. When also $\delta_2 \to \infty$, second period price, $p_2$, will be fully revealing, so

[10] Foster and Viswanathan (1996) is a notable exception. For a detailed review of the literature, see Brunnermeier (2001).

Londoners will behave as if they could liquidate for $\theta_w$, which is the only relevant factor remaining. Hence, in the case of $\kappa, \delta_2 \to \infty$, effectively we have a two period model with one factor as in Brown and Jennings. When $\delta_2$ is finite, the model resembles to He and Wang (1995) as the liquidation value for Londoners, $p_2$, will contain the random term $u_2$ as well. However, when only $\omega \to \infty$, we have a very different structure, where the private information sets of Londoners and New Yorkers get separated.

## 3.2 Analytical results

In the first part of this section, we show that an equilibrium of the present model exists. In the second part, we present results on the equilibrium volume.

### 3.2.1 Equilibrium and existence

We search for a linear equilibrium, so we assume that prices are given by the functions

$$\begin{aligned} p_2 &= c_2 y + b_2 (\theta_s + \theta_w) + f_2 q_1 + g_2 q_0 - e_2 u_2 \\ p_1 &= c_1 y + b_1 (\theta_k + \theta_w) + f_1 q_0 - e_1 u_1 \\ p_0 &= b_0 (\theta_k + \theta_w) - e_0 u_0 \end{aligned} \tag{3}$$

where $c_t, b_t, e_t, f_1, f_2, g_2$ are undetermined coefficients, while $q_1, q_0$ are specified below. Prices together with past prices and the public information are informationally equivalent with the following price signals

$$\begin{aligned} q_2 &= \frac{1}{b_2}(p_2 - c_2 y - f_2 q_1 - g_2 q_0) = (\theta_s + \theta_w) - \frac{e_2}{b_2} u_2 \\ q_1 &= \frac{1}{b_1}(p_1 - c_1 y - f_1 q_0) = (\theta_k + \theta_w) - \frac{e_1}{b_1} u_1 \\ q_0 &= \frac{1}{b_0} p_0 = (\theta_k + \theta_w) - \frac{e_0}{b_0} u_0. \end{aligned} \tag{4}$$

Below, we will show that the equilibrium trading activity is determined only by the noisiness of these price signals, so we define $\tau_t^2$ as the precision of $q_t$:

$$\frac{1}{\tau_t^2} = \frac{e_t^2}{b_t^2 \delta_t^2} \text{ and } \frac{\tau_t}{\delta_t} = \frac{b_t}{e_t}.$$

Let us also define the following coefficients of variables in traders' information sets in the different conditional expectations of New Yorkers and Londoners:

$$E(\theta | z_j, y, q_2, q_1, q_0) = \bar{b} z_j + \bar{c} y + \bar{e} q_2 + \bar{f} q_1 + \bar{g} q_0 \tag{5}$$

$$E(\theta_s + \theta_w | x_i, y, q_1, q_0) = b_s x_i + c_s y + e_s q_1 + f_s q_0 \tag{6}$$

$$E(\theta | x_i, q_0) = E(\theta_k + \theta_w | x_i, q_0) = E(q_1 | x_i, q_0) = E(y | x_i, q_0) = b_y x_i + e_y q_0. \tag{7}$$

From standard results(e.g. Brown and Jennings, 1989), we know that the demand functions of New Yorkers in period 2 and Londoners in period 1 will be

$$d_2^j = \frac{E\left(\theta|z_j, y, p_2, q_1, q_0\right) - p_2}{avar\left(\theta|z_j, y, p_2, q_1, q_0\right)} = \frac{\bar{c}y + \bar{b}x_j + \bar{f}q_1 + \bar{e}q_2 + \bar{g}q_0 - p_2}{avar\left(\theta|z_j, y, p_2, q_1, q_0\right)} \tag{8}$$

$$d_1^i = \frac{E\left(p_2|x_i, y, p_1, q_0\right) - p_1}{avar\left(p_2|x_i, y, p_1, q_0\right)} = \frac{c_2 y + b_2\left(b_s x_i + c_s y + e_s q_1 + f_s q_0\right) + f_2 q_1 + g_2 q_0 - p_1}{avar\left(p_2|x_i, y, p_1, q_0\right)} \tag{9}$$

Finding the demand function in period 0 is a bit more subtle. Londoners maximize the following expected utility in period 0:

$$E\left(-\exp\left(-a\left(p_1 - p_0\right)d_1 - \frac{E\left(p_2|q_1, y, x_j, q_0\right) - p_1}{avar\left(p_2|q_1, y, x_j, q_0\right)}\left(p_2 - p_1\right)\right)|x_i, q_0\right).$$

The source of the difficulty is that there are two random variables in this expression, $p_1$ (or $q_1$) and $y$ In the appendix, we show that as the outcome of this maximization – the demand function in period 0 – is

$$\begin{aligned} d_0^i &= \frac{\left(E\left(p_1|x_i, q_0\right) - p_0\right)\left(\sigma_q s + 1\right)}{a\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right)} + \\ &+ \frac{b_s E\left(d_1^i|x_i, q_0\right)\left(c_1\sigma_{yq} + b_1\sigma_q\right)}{a\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right)} \end{aligned} \tag{10}$$

where

$$\begin{pmatrix} \sigma_y & \sigma_{yq} \\ \sigma_{yq} & \sigma_q \end{pmatrix} = var\left(\begin{pmatrix} y \\ q_1 \end{pmatrix}|x_i, q_0\right).$$

is the variance-covariance matrix of $y$ and $q_1$ conditional on a London-trader's information set in period 0. Intuitively, the first part in expression (10) represents the short-term demand component, while the second part represents the hedging component for demand in period 1.

We can show that in equilibrium, demand functions can be characterized completely by the equilibrium values of $\tau_t$ in the following manner:

$$d_2^j = \frac{\tau_2}{\delta_2}(z_j - q_2) = \frac{\tau_2}{\delta_2}\varepsilon_j + u_2 \tag{11}$$

$$d_1^i = \frac{\tau_1}{\delta_1}\left(x_i - q_1\right) = \frac{\tau_1}{\delta_1}\varepsilon_i + u_1 \tag{12}$$

$$d_0^i = \frac{\tau_0}{\delta_0}\left(x_i - q_0\right) = \frac{\tau_0}{\delta_0}\varepsilon_i + u_0 \tag{13}$$

The right hand sides of the three equations show that in each period, total positions consist of two parts. There is a risk-sharing part, $u_t$, which is purchased by each agent regardless of her information, and there is a speculative part, $\frac{\tau_t}{\delta t}\varepsilon_i$, which depends on the difference between the agent's signal and the true value of the factor, $\varepsilon_i$ or $\varepsilon_j$, and the fraction $\frac{\tau_t}{\delta t}$. It is apparent that $\frac{\tau_t}{\delta t}$ determines how intensively the trader uses her private information to bet against the others, so we will label this fraction as trading intensity in period $t$. Here, we only present the steps which lead to (11). Expressions (12) and (13) are obtained very similarly.

From (8), the market clearing condition is

$$D_2 = \frac{\bar{c}y + \bar{b}\left(\theta_s + \theta_w\right) + \bar{f}q_1 + \bar{e}q_2 + \bar{g}q_0 - p_2}{avar\left(\theta | z_j, y, p_2, q_1, q_0\right)} = u_2.$$

Using (4) and rearranging gives

$$\bar{c}y + \bar{b}\left(\theta_s + \theta_w\right) + \bar{f}q_1 + \bar{e}q_2 + \bar{g}q_0 - avar\left(\theta | z_j, y, p_2, q_1, q_0\right)u_2 = b_2 q_2 + c_2 y + f_2 q_1 + g_2 q_0.$$

As the two sides has to be equal in equilibrium for any realizations of $u_1, u_2$ and $\eta$,

$$\begin{aligned} c_2 &= \bar{c} && (14)\\ f_2 &= \bar{f} && (15)\\ g_2 &= \bar{g} && (16) \end{aligned}$$

must hold. This implies

$$\bar{b}\frac{e_2}{b_2}\left(\theta_s + \theta_w\right) - avar\left(\theta | z_j, y, p_2, q_1, q_0\right)\frac{e_2}{b_2}u_2 = \left(b_2 - \bar{e}\right)\frac{e_2}{b_2}q_2.$$

As $q_2 = \theta_s + \theta_w - \frac{e_2}{b_2}u_2$, this gives us

$$\frac{\bar{b}}{avar\left(\theta | z_j, y, p_2, q_1, q_0\right)} = \frac{b_2}{e_2} = \frac{\left(b_2 - \bar{e}\right)}{avar\left(\theta | z_j, y, p_2, q_1, q_0\right)},$$

consequently,

$$b_2 = \bar{b} + \bar{e}. \tag{17}$$

Note, that expressions (14)-(16) and (17) determine the equilibrium value of the coefficients in the price function,(3), in terms of coefficients of the conditional expectation of $\theta$.

If we substitute out $p_2, c_2, g_2, f_2$ from the left hand side of (8), we get

$$E\left(\theta | z_j, y, p_2, q_1, q_0\right) - p_2 = b_2\left(z_j - q_2\right),$$

which – together with the definition of $\tau_2$ and $q_2$ – implies

$$d_2^i = \frac{\bar{b}\left(z_j - q_2\right)}{avar\left(\theta | z_j, y, p_2, q_1, q_0\right)} = \frac{\tau_2}{\delta_2}(z_j - q_2) = \frac{\tau_2}{\delta_2}\varepsilon_j + u_2. \tag{18}$$

Very similar steps applied to (9) and (10) gives us

$$d_1^i = \frac{1}{ab_2}\frac{b_s}{var_i\left(\theta_s+\theta_k\right)+\frac{1}{\tau_2^2}}\left(x_i-q_1\right)=\frac{\tau_1}{\delta_1}\left(x_i-q_1\right)=\frac{\tau_1}{\delta_1}\varepsilon_i+u_1 \tag{19}$$

$$d_0^i = \frac{\left(\sigma_q s+1\right)\left(\left(c_1+b_1\right)b_y+s\left(1-b_y\right)\frac{c_1\sigma_{yq}+b_1\sigma_q}{\sigma_q s+1}\right)}{a\left(c_1^2\sigma_y+c_1^2 s\sigma_y\sigma_q-c_1^2 s\sigma_{yq}^2+\sigma_q b_1^2+2c_1\sigma_{yq}b_1\right)}\left(x_i-q_0\right)=\frac{\tau_0}{\delta_0}\left(x_i-q_0\right)=\frac{\tau_0}{\delta_0}\varepsilon_i+u_0 \tag{20}$$

From this procedure – similarly to expressions (14)-(16) and (17) – we also gain expressions for $c_1$ and $b_1$ in terms of coefficients of the conditional expectations (5)-(7)[11]. All of these, together with the expectational coefficients are given in the appendix. The last step is to find the equilibrium trading intensities,$\frac{\tau_t}{\delta_t}$, which give the equilibrium demand functions. For this, we simply plug in the expressions for $b_s,\bar{b}$,$b_2$ and the conditional variances into (18) and (19) and equate the coefficients of $(z_j-q_2),(x_i-q_1)$ and $(x_i-q_0)$ in the two sides of the equations (18)-(20). This gives a system of three equations with the three unknowns of $\tau_1,\tau_2,\tau_0$ :

$$\begin{aligned}
\tau_2 &= f^2\left(\tau_2,\tau_1,\tau_0\right)= \qquad (21)\\
&= \delta_2\frac{1}{a}\alpha\frac{\tau_0^2\omega+\tau_1^2\omega+\kappa\omega+\tau_0^2\kappa+\tau_1^2\kappa+\kappa^2}{\tau_0^2\kappa+\tau_0^2\tau_2^2+\tau_0^2\omega+\alpha\tau_0^2+\alpha\tau_1^2+\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^2+\omega\tau_2^2+\tau_1^2\tau_2^2+\tau_1^2\omega+\tau_1^2\kappa}\\
\tau_1 &= f^1\left(\tau_2,\tau_1,\tau_0\right)= \qquad (22)\\
&= \delta_1\tau_2^2\alpha\frac{\kappa^2-\omega\beta}{a\left(\kappa\tau_0^2\tau_2^2+\kappa\omega\tau_2^2+\tau_2^2\kappa^2+\omega\tau_1^2\tau_2^2+2\alpha\omega\tau_2^2+\kappa\tau_1^2\tau_2^2+2\kappa\alpha\tau_2^2+\tau_0^2\omega\tau_2^2+\alpha\kappa^2+\kappa\omega\alpha\right)}\\
\tau_0 &= f^0\left(\tau_2,\tau_1,\tau_0\right)=\frac{\delta_0\left(\sigma_q s+1\right)\left(c_1+b_1\right)b_y+s\left(1-b_y\right)\left(c_1\sigma_{yq}+b_1\sigma_q\right)}{a\left(c_1^2\sigma_y+c_1^2 s\sigma_y\sigma_q-c_1^2 s\sigma_{yq}^2+\sigma_q b_1^2+2c_1\sigma_{yq}b_1\right)}. \qquad (23)
\end{aligned}$$

Note, that all the building-blocks of $f^0\left(\tau_2,\tau_1,\tau_0\right)$ – which are $\sigma_q,\sigma_y,\sigma_{yq},b_y,c_1,b_1$ and $s$ – can be expressed as functions of $\tau_2,\tau_1,\tau_0$ only (see appendix). Furthermore, it is easy to see that the corresponding equilibrium intensities when there is no announcement will be given by the same equations by setting $\beta=0$. When it could cause misunderstanding, we will distinguish between $\tau_2,\tau_1,\tau_0$ of the announcement and the no-announcement cases by the subscript $n$, for no-announcement.

Hence, the equilibrium exists if and only if this system of equations has a fix point. The following proposition states that this will be the case for any parameter values.

**Theorem 1 (Existence)** *From (21)-(23) any equilibrium is a fixed-point of a system:*

$$\tau_2=f^2\left(\tau_2,\tau_1,\tau_0\right),\qquad \tau_1=f^1\left(\tau_2,\tau_1,\tau_0\right),\qquad \tau_0=f^0\left(\tau_2,\tau_1,\tau_0\right)$$

*There is always at least one equilibrium of this system both for the announcement and the no-announcement cases.*

**Proof.** The proof is in the Appendix. ■

[11] Actually, we also obtain similar expressions for the other coefficients in the price functions – $e_2,f_1,e_1,b_0,e_0$ – but as they are not relevant for our purposes, we omit them to save space.

### 3.2.2 Announcement and volume

As the focus of this paper is the effect of announcement on trading volume, we will be interested in the change of volume in period 1 due to the announcement. From equations (9) and (10), the amount of trading of trader $i$ in period 1 will be given by

$$v_1^i = d_1^i - d_0^i = \left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i - u_0 + u_1.$$

Just as total positions, the total amount of trade of individual $i$ consist of two parts. There is an information-independent risk-sharing part, $u_1 - u_0$, and there is a speculative part $\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i$, which is determined by the difference of trading intensities in the two periods and the private information of the trader. If we aggregate across traders, we get the following expression for total volume in period 1:

$$\begin{aligned}V_1 = \frac{1}{2}\int\left|d_1^i - d_0^i\right| di = \frac{1}{2}\int_{\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i > u_0 - u_1}\left(\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i + u_1 - u_0\right)\phi\left(\alpha\varepsilon_i\right)d\varepsilon_i - \\ -\frac{1}{2}\int_{\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i < u_0 - u_1}\left(\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i + u_1 - u_0\right)\phi\left(\alpha\varepsilon_i\right)d\varepsilon_i = \\ = \left|\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\right|\frac{1}{\sqrt{\alpha}}\phi\left(T\right) + sgn\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\left(u_1 - u_0\right)\frac{1}{2}\left(1 - 2\Phi\left(T\right)\right)\end{aligned}$$

with $T = \alpha\frac{u_0 - u_1}{\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)}$,where we used the result that if $\zeta \sim N\left(\mu, \sigma^2\right)$, then

$$\int_{\zeta > L}\zeta\frac{\phi\left(\frac{\zeta - \mu}{\sigma}\right)}{\Phi\left(\alpha\right)}d\zeta = E\left(\zeta|\zeta > L\right) = \mu + \sigma\lambda\left(\alpha\right)$$

with $\lambda\left(\alpha\right) = \frac{\phi(\alpha)}{1 - \Phi(\alpha)}$ and $\alpha = \frac{L - \mu}{\sigma}$.

Hence, the aggregate volume depends only on the realization of $u_1 - u_0$, the precision of the private signals $\alpha$, and the distance between trading intensities, $\left|\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\right|$. As the first one is unrelated to information or announcements, we will focus on the speculative volume, which we define as the volume when there is no risk-sharing trade:

$$V_1^S = V_1|_{u_0 = u_1} = \left|\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right|\frac{1}{\sqrt{\alpha 2\pi}}.$$

It must be clear now that – for results on the effect of announcement on speculative volume – we only have to compare the change in trading intensities in the announcement case, $\left|\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right|$, and the no-announcement case, $\left|\frac{\tau_1^n}{\delta_1} - \frac{\tau_0^n}{\delta_0}\right|$. The main result of this paper that the outcome of this comparison will depend heavily and systematically on the information structure i.e. on the relative importance of the common factor, $\theta_w$, and the individual factors $\theta_s, \theta_k$. The following proposition shows that as the individual factors are getting less important and the common factor becomes more important, volume disappears. This result is in line with our earlier observation that in a rational model with one factor trading volume around announcements is small, because the effects of increasing precision of opinions and decreasing disagreement cancel out.

**Proposition 4** *As* $\delta_2, \kappa \to \infty$

$$\frac{\tau_2}{\delta_2} = \frac{\tau_1}{\delta_1} = \frac{\tau_0}{\delta_0} = \frac{\tau_2^n}{\delta_2} = \frac{\tau_1^n}{\delta_1} = \frac{\tau_0^n}{\delta_0} = \frac{\alpha}{a}$$

, *hence* $V_1 = V_1^n = 0$ *in this limit.*

**Proof.** For $\frac{\tau_2}{\delta_2}, \frac{\tau_1}{\delta_1}, \frac{\tau_2^n}{\delta_2}$ and $\frac{\tau_1^n}{\delta_1}$, the result comes from the simple observation that the ordered limit

$$\lim_{\delta_2 \to \infty} \lim_{\kappa \to \infty} f^2(\tau_2, \tau_1, \tau_0) = \frac{\alpha}{a}$$

and after substitution of $\tau_2 = \frac{\alpha}{a}$

$$\lim_{\delta_2 \to \infty} \lim_{\kappa \to \infty} f^1(\tau_2, \tau_1, \tau_0) = \frac{\alpha}{a}.$$

The result for $\frac{\tau_0}{\delta_0}$ and $\frac{\tau_0^n}{\delta_0}$ can be obtained in a similar, but much more tedious way, if we take the limit of all the building-blocks of $f^0(\tau_2, \tau_1, \tau_0)$ and plug them in. ∎

We confront this result with the next proposition, where we show that if one measures the effect of announcement by the proportion of volume in the announcement case to volume in the no-announcement case, this proportion will be arbitrary large as the common factor, $\theta_w$, loses its importance. The same is true for the proportion of speculative positions in both periods.

**Proposition 5** *If* $\omega$ *is large enough* $D_1^s = \frac{1}{2}\int \left|\frac{\tau_1}{\delta_0}\varepsilon_i\right| di > \frac{1}{2}\int \left|\frac{\tau_1^n}{\delta_0}\varepsilon_i\right| di = D_1^n$ *and* $V_1 > V_1^n$ *and as* $\omega \to \infty$ *,*$\frac{D_1}{D_1^n} \to \infty$ *,*$\frac{D_0}{D_0^n} \to \infty$ *. Furthermore,* $\frac{V_1}{V_1^n} \to \infty$ *(for almost all parameters).*

**Proof.** The proof is in the appendix. ∎

The intuition of this result goes as follows. Let us suppose that $\omega$ is large, so the common factor, $\theta_w$ is unimportant i.e. only the individual factors, $\theta_s$ and $\theta_k$ matter. Traders can bet only on those variables which are not part of the public information set. From the Londoners point of view in period 1 (the second period when they trade), the only variable in $p_2$ which is not part of the public information set – apart from the noise, $u_2$ – is $\theta_s$. But first period traders have no information on $\theta_s$, only on $\theta_k$. Hence, they will agree that they do not know anything (i.e. there guess will be the a priori mean, 0), and there will be agreement and no speculative trade. But if Londoners do not trade on their private information, their private information cannot be channeled into prices, so $p_1$ will be pure noise. So if we go one period back, in period 0, Londoners should bet on $p_1$ and $p_2$, but they do not have any information neither on $p_1$, as it is pure noise , nor on $p_2$, because they do not know anything about $\theta_s$. Hence, there will be no speculative trade in period 0 either. It means that the speculative volume, the difference between individual speculative positions in period 0 and period 1, will also be zero. However, with public announcement the situation changes. The public announcement is $y \approx \theta_s + \theta_k + \eta$, if $\theta_w$ is unimportant. So Londoners will have some information on the sum of $\theta_s$ and

Figure 1: Trading intensities in periods 0 and 1 in the cases of announcement and no-announcement (the coefficient of $\varepsilon_i$ in the expression $d_t^i$) as the information set of traders across groups becomes more separated i.e. $\omega$ increases. Parameter values are $\delta_2 = \kappa = 5$, $\alpha = \beta = \delta_1 = \delta_0 = 1$.

$\theta_k$. But together with their private information on $\theta_k$, it gives them some information on $\theta_s$. What is more, as they have different guesses on $\theta_k$ due to their different private signals, their guesses on $\theta_s$ will also be different. Therefore, public announcement increases disagreement. With the opposite logic as in the no-announcement case, there will be trade in all periods and there will be volume.

### 3.3 Numerical Results

We calculated numerically the fix point of the system (21)-(23) with several parameter combinations. A typical graph of trading intensities $\frac{\tau_t}{\delta_t}$ and $\frac{\tau_t^n}{\delta_t}$ is shown in Figure 1. The middle two lines are $\frac{\tau_0^n}{\delta_0}$ and $\frac{\tau_1^n}{\delta_1}$ (the trading intensity in the no-announcement case in period 1 and 0). Both goes to 0 as $\omega \to \infty$ as it is stated in the proof of Proposition 5 and in line with our intuitive story when $\theta_w$ is unimportant. With announcement, in period 0 traders bet intensively on the size of $y$ based on their private information (line with asterisk), and in period 1 they bet intensively on $p_2$ (solid line). The larger the distance between the two lines, the larger the trading volume around announcements. When $\omega$ is small, we are close to the standard information case (one factor model). It is apparent that at this extreme, all lines are almost equal, so trading volume is small. If $\kappa$ and $\delta_2$ would be large enough, all lines would coincide as it is stated in Proposition 4. It is spectacular that as $\omega$ grows – private information sets become separated across groups – the lines corresponding to the announcement cases fan out. This shows the potential of our story in explaining the jump in trading volume around announcements.

# 4 Confrontation with empirical results

## 4.1 Established stylized facts

Recent evidence from high-frequency data-sets (Evans and Lyons, 2001,2003, Love and Payne, 2003, Love 2004, Fleming and Remolona, 1999) show a prolonged intense trading period after announcements when

1. there is a simultaneous increase in buying orders and selling orders[12]
2. order flows, the difference between buying and selling orders, are more volatile
3. order flows are more informative i.e. they influence price formation more.

Our rational expectations model is of a reduced form. All traders submit whole demand schedules and orders are executed on the market clearing price. Hence, in our model there is no order flow. In reality, a market maker neither observes whole demand schedules nor observes all of them at the same time. She has to map them through time. She quotes a price which is good for any amount, some traders make transactions on this price, and the market maker updates her quote depending on the received orders. Order flow is like the aggregate trades of a small group of traders executed at a close-to-equilibrium price. We believe that the nearest we can get in our model to the behaviour of order flow, is to consider the behaviour of individual orders at equilibrium price, because the aggregation of a small number of trades must have similar characteristics to the parts of this aggregation. We will argue that if one is willing to accept individual trades as a proxy for order flow, then our model is consistent with the three observed facts above.

1. It is easy to see that both buys and sells increase due to announcement in our model as individual orders are given by

$$d_1^i - d_0^i = \left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i + u_1 - u_0 = \left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)x_i - \left(\frac{\tau_1}{\delta_1}q_1 - \frac{\tau_0}{\delta_0}q_0\right)$$

   and we showed that $\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)$ can be much larger when there is announcement, than when there is none.

2. Similarly, the volatility of individual orders depend positively on $\left|\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right|$ as well, since

$$var\left(d_1^i - d_0^i\right) = \left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)^2 \frac{1}{\alpha} + \frac{1}{\delta_1^2} + \frac{1}{\delta_2^2}$$

   .

[12] In the FX market, all dealers act as market makers, so all submit bid and ask prices simultaneously to the brokerage system, which are good for any amount. Then any of the dealers can initiate transactions on the best submited quotes. A transaction is called a buying (selling) order, if the initiator of the transaction buys (sells) the comodity currency. Hence, the number of buys and the number of sells in any FX dataset can move independently of each other. (See Lyons, 2001, for detailed discussion on the microstructure of FX markets.)

3. Note, that in the mapping process of demand curves described above, the information content of individual orders depends on how strongly they are correlated with private information. It is so, because the more heavily a trader uses her information on her speculative betting, the easier the market maker can deduct the private information of the trader. Since for a given price, $q_1$,

$$cov\left(d_1^i - d_0^i, x_i\right) = \left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right).$$

The informativeness of prices depends again only on $\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)$.

Evans and Lyons (2001) argue that the finding of increased informative trade due to announcements is inconsistent with a common-priors model:

> "...when (1) information is publicly observed and (2) all market participants agree on the mapping from that information to price, then price adjustment occurs independently of order flow. Our finding that the adjustment of price to announcements depends on order flow suggests a violation of the second condition that all participants agree on the mapping."

Although in our model (1) and (2) is true, there is a lot of price adjustment through trade. However, it is not the public information, which is built in this way, but the existing private information becomes relevant and flows into the market.

# 5 Conclusion

We showed that common knowledge public announcements can move opinions further apart, when agents are rational and the model is common knowledge. The main intuition behind our model is that the implication of public information can be very different if it is coupled with different private information sets. We showed that this fact can cause large, volatile and informative trading volume around public announcements. We had the following critical assumptions. We assumed that there were some early traders (Londoners) who had to resale the asset to new entrants (New Yorkers). We allowed for weaker correlation between private information sets across two groups, and it was necessary that public announcement was not independent of any of the information sets. In the second half of the paper we argued that implications are largely consistent with empirical observations.

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# Appendix

## A.1 Demand in period 0

In period zero traders maximize the expected utility

$$E\left(-\exp\left(-a\left(p_1-p_0\right)d_0^i-\frac{E\left(p_2|q_1,y,x_i,q_0\right)-p_1}{avar\left(p_2|q_1,y,x_i,q_0\right)}\left(p_2-p_1\right)\right)|x_i,q_0\right)=$$
$$=E\left(E\left(-\exp\left(-a\left(p_1-p_0\right)d_0^i-\frac{E\left(p_2|q_1,y,x_i,q_0\right)-p_1}{avar\left(p_2|q_1,y,x_i,q_0\right)}\left(p_2-p_1\right)\right)q_0,y,x_i,q_1\right)|x_i,q_0\right)=$$
$$=E\left(-\exp\left(-a\left(p_1-p_0\right)d_0^i-\frac{\left(E\left(p_2|q_1,y,x_i,q_0\right)-p_1\right)^2}{2var\left(p_2|q_1,y,x_j,q_0\right)}\right)|x_i,q_0\right)=$$
$$=E\left(-\exp\left(-a\left(b_1q_1+c_1y+f_1q_0-p_0\right)d_0^i-\frac{1}{2}s\left(x_i-q_1\right)^2\right)|x_i,q_0\right)$$

where

$$s=\frac{b_s^2}{\left(var\left(\theta_s+\theta_w|q_1,y,x_j,q_0\right)+\frac{1}{\tau_2^2}\right)}=b_2a\frac{\tau_1}{\delta_1}b_s.$$

If we write the expression in the inner bracket into matrix form and we use the standard result for the expectation of exponentials with quadratic forms[13], we get

$$E\left(-\exp\left(\begin{array}{c}-\frac{1}{2}sx_i^2-a\left(f_1q_0-p_0\right)d_0^i+\begin{pmatrix}-ac_1d_0^i & \left(-ab_1d_0^i+sx_i\right)\end{pmatrix}\begin{pmatrix}y\\ q_1\end{pmatrix}\\ -\begin{pmatrix}y & q_1\end{pmatrix}\left(\frac{1}{2}s\right)\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}y\\ q_1\end{pmatrix}\end{array}\right)|x_i,q_0\right)=$$

$$=-\exp\left(\begin{array}{c}-\frac{1}{2}sx_i^2-a\left(f_1q_0-p_0\right)d_0^i+\begin{pmatrix}-ac_1d_0^i & \left(-ab_1d_0^i+sx_i\right)\end{pmatrix}\begin{pmatrix}\mu_y\\ \mu_q\end{pmatrix}+\\ \frac{1}{2}\left(\begin{pmatrix}-ac_1d_0^i & \left(-ab_1d_0^i+sx_i\right)\end{pmatrix}-2\begin{pmatrix}\mu_y & \mu_q\end{pmatrix}\begin{pmatrix}0 & 0\\ 0 & \frac{1}{2}s\end{pmatrix}\right)\\ \left(\begin{pmatrix}0 & 0\\ 0 & s\end{pmatrix}+\begin{pmatrix}\sigma_y & \sigma_{yq}\\ \sigma_{yq} & \sigma_q\end{pmatrix}^{-1}\right)^{-1}\left(\begin{pmatrix}-ac_1d_0^i\\ -ab_1d_0^i+sx_i\end{pmatrix}-2\begin{pmatrix}0 & 0\\ 0 & \frac{1}{2}s\end{pmatrix}\begin{pmatrix}\mu_y\\ \mu_q\end{pmatrix}\right)\\ -\begin{pmatrix}\mu_y & \mu_q\end{pmatrix}\begin{pmatrix}0 & 0\\ 0 & \frac{1}{2}s\end{pmatrix}\begin{pmatrix}\mu_y\\ \mu_q\end{pmatrix}\end{array}\right)$$

[13] If $c$ is constant scalar, $L$ is a $nx1$ constant vector, $N$ is an $nxn$ constant matrix and $M$ is an $nx1$ stochastic matrix and $I$ is an information set, then

$$E\left(-\exp\left(c+L'M-M'NM'\right)|I\right)=$$
$$-\left|W\right|^{-1/2}\left|2N+W^{-1}\right|^{-1/2}\exp\left(c+L'Q-Q'NQ+\frac{1}{2}\left(L'-2Q'N\right)\right)\left(2N+W^{-1}\right)^{-1}\left(L-2NQ\right)\Big)$$

where $Q=E\left(M|I\right)$ and $W=var\left(M|I\right)$ (see Brunnermeier, 2001, page 110).

where

$$\begin{aligned}\mu_q &= E\left(q_1|x_i, q_0\right)\\ \mu_y &= E\left(y|x_i, q_0\right).\end{aligned}$$

and

$$\begin{pmatrix}\sigma_y & \sigma_{yq}\\ \sigma_{yq} & \sigma_q\end{pmatrix} = var\left(\begin{pmatrix} y\\ q_1\end{pmatrix}|x_i, q_0\right).$$

Maximizing the term in the bracket with respect to $d_0^i$ gives the demand function

$$d_0^i = \frac{\left(c_1\mu_y + \mu_q b_1 + q_0 f_1 - p_0\right)\left(\sigma_q s + 1\right)}{a\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right)} + \frac{s\left(x_i - \mu_q\right)\left(c_1\sigma_{yq} + b_1\sigma_q\right)}{a\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right)}$$

, which is identical with the one in the text. The second order condition of the maximization is

$$\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right) > 0.$$

## A.2 Expectations, variances and coefficients in the price-functions

We give here the conditional expectations and variances obtained by standard results on normal variables (see e.g. Brunnermeier, 2001, p12). We also give the equilibrium expressions for coefficients in the price function. The method to obtain the latter is described in the text.

$$\begin{aligned}\sigma_y &= \frac{2\kappa\omega\beta + \kappa^2\beta + \beta\alpha\omega + \beta\alpha\kappa + \beta\tau_0^2\omega + \beta\tau_0^2\kappa + \kappa\alpha\omega + \kappa^2\alpha + \kappa\tau_0^2\omega + \kappa^2\tau_0^2 + \kappa^2\omega}{\left(\alpha\omega + \kappa\alpha + \tau_0^2\omega + \kappa\tau_0^2 + \kappa\omega\right)\kappa\beta}\\ \sigma_q &= \frac{\tau_1^2\omega + \tau_1^2\kappa + \alpha\omega + \kappa\alpha + \tau_0^2\omega + \kappa\tau_0^2 + \kappa\omega}{\left(\alpha\omega + \kappa\alpha + \tau_0^2\omega + \kappa\tau_0^2 + \kappa\omega\right)\tau_1^2}\\ \sigma_{yq} &= \frac{\omega + \kappa}{\alpha\omega + \kappa\alpha + \tau_0^2\omega + \kappa\tau_0^2 + \kappa\omega}\\ b_y &= \left(\omega + \kappa\right)\frac{\alpha}{\alpha\omega + \alpha\kappa + \tau_0^2\omega + \tau_0^2\kappa + \kappa\omega}\\ c_1 &= \left(b_2 c_s + \bar{c}\right)\\ b_1 &= \left(b_2\left(b_s + e_s\right) + \bar{f}\right)\end{aligned}$$

where
$b_2 = \frac{\left(\alpha\omega\tau_0^2 + \alpha\omega\tau_1^2 + \kappa\omega\alpha + \kappa\alpha\tau_0^2 + \kappa\alpha\tau_1^2 + \alpha\kappa^2 + \tau_0^2\omega\tau_2^2 + \tau_1^2\omega\tau_2^2 + \kappa\omega\tau_2^2 + \kappa\tau_0^2\tau_2^2 + \kappa\tau_1^2\tau_2^2 + \tau_2^2\kappa^2\right)}{B}$
with

$$B = \begin{pmatrix}\alpha\omega\beta + \kappa^2\beta + \tau_1^2\kappa^2 + \tau_2^2\kappa^2 + \alpha\kappa^2 + \kappa^2\omega + \tau_0^2\kappa^2 + \alpha\kappa\beta + \kappa\omega\tau_1^2 + \tau_1^2\omega\tau_2^2 +\\ \kappa\omega\tau_2^2 + 2\kappa\tau_0^2\tau_2^2 + \tau_0^2\omega\tau_2^2 + 2\kappa\tau_1^2\tau_2^2 + \kappa\omega\tau_0^2 + \kappa\omega\alpha +\\ +\tau_0^2\kappa\beta + \tau_1^2\omega\beta + \tau_1^2\kappa\beta + \tau_0^2\omega\beta + \tau_2^2\beta\kappa + \tau_1^2\tau_2^2\beta + \tau_0^2\tau_2^2\beta + \tau_2^2\omega\beta + 2\kappa\omega\beta\\ +2\kappa\alpha\tau_0^2 + 2\kappa\alpha\tau_1^2 + \alpha\omega\tau_1^2 + \alpha\tau_1^2\beta + \alpha\omega\tau_0^2 + \alpha\tau_0^2\beta\end{pmatrix}$$

and

$\bar{f} = \frac{1}{B}\tau_1^2\left(\alpha + \tau_2^2 + \kappa\right)(\omega + \kappa)$

$\bar{c} = \frac{\beta\left(\tau_0^2\kappa+\tau_0^2\tau_2^2+\tau_0^2\omega+\alpha\tau_0^2+\alpha\tau_1^2+\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^2+\omega\tau_2^2+\tau_1^2\tau_2^2+\tau_1^2\omega+\tau_1^2\kappa\right)}{B}$

$c_s = \frac{\beta\left(\kappa+\tau_1^2+\alpha+\tau_0^2\right)(\omega+\kappa)}{\kappa^2\omega+2\kappa\omega\beta+\kappa\omega\tau_0^2+\tau_0^2\kappa^2+\kappa^2\beta+\alpha\kappa^2+\kappa\omega\alpha+\kappa\omega\tau_1^2+\tau_1^2\kappa^2+\tau_0^2\kappa\beta+\tau_1^2\omega\beta+\tau_1^2\kappa\beta+\tau_0^2\omega\beta+\alpha\kappa\beta+\alpha\omega\beta}$

$e_s = \frac{\tau_1^2\kappa^2-\tau_1^2\omega\beta}{\kappa^2\omega+2\kappa\omega\beta+\kappa\omega\tau_0^2+\tau_0^2\kappa^2+\kappa^2\beta+\alpha\kappa^2+\kappa\omega\alpha+\kappa\omega\tau_1^2+\tau_1^2\kappa^2+\tau_0^2\kappa\beta+\tau_1^2\omega\beta+\tau_1^2\kappa\beta+\tau_0^2\omega\beta+\alpha\kappa\beta+\alpha\omega\beta}$

$b_s = \frac{\alpha\kappa^2-\alpha\omega\beta}{\kappa^2\omega+2\kappa\omega\beta+\kappa\omega\tau_0^2+\tau_0^2\kappa^2+\kappa^2\beta+\alpha\kappa^2+\kappa\omega\alpha+\kappa\omega\tau_1^2+\tau_1^2\kappa^2+\tau_0^2\kappa\beta+\tau_1^2\omega\beta+\tau_1^2\kappa\beta+\tau_0^2\omega\beta+\alpha\kappa\beta+\alpha\omega\beta}$

$var\left(\theta_s + \theta_w | x_i, q_1, q_0, y\right) =$

$= \frac{\tau_1^2\omega+2\tau_1^2\kappa+\omega\beta+\kappa^2+\kappa\omega+\alpha\omega+2\alpha\kappa+\tau_1^2\beta+\alpha\beta+2\tau_0^2\kappa+\tau_0^2\omega+\beta\kappa+\tau_0^2\beta}{\kappa^2\omega+2\kappa\omega\beta+\kappa\omega\tau_0^2+\tau_0^2\kappa^2+\kappa^2\beta+\alpha\kappa^2+\kappa\omega\alpha+\kappa\omega\tau_1^2+\tau_1^2\kappa^2+\tau_0^2\kappa\beta+\tau_1^2\omega\beta+\tau_1^2\kappa\beta+\tau_0^2\omega\beta+\alpha\kappa\beta+\alpha\omega\beta}$

$var\left(\theta | z_j, q_2, q_1, q_0, y\right) = \frac{\left(\tau_0^2\kappa+\tau_0^2\tau_2^2+\tau_0^2\omega+\alpha\tau_0^2+\alpha\tau_1^2+\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^2+\omega\tau_2^2+\tau_1^2\tau_2^2+\tau_1^2\omega+\tau_1^2\kappa\right)}{B}$

## A.3 Proof of existence

The equilibrium is given by the fixed point of the system

$$
\begin{aligned}
\tau_2 &= f^2\left(\tau_2, \tau_1, \tau_0\right) = \\
&= \delta_2 \frac{1}{a}\alpha \frac{\tau_0^2\omega + \tau_1^2\omega + \kappa\omega + \tau_0^2\kappa + \tau_1^2\kappa + \kappa^2}{\tau_0^2\kappa + \tau_0^2\tau_2^2 + \tau_0^2\omega + \alpha\tau_0^2 + \alpha\tau_1^2 + \alpha\omega + \kappa\alpha + \kappa^2 + 2\kappa\omega + \kappa\tau_2^2 + \omega\tau_2^2 + \tau_1^2\tau_2^2 + \tau_1^2\omega + \tau_1^2\kappa} \\
\tau_1 &= f^1\left(\tau_2, \tau_1, \tau_0\right) = \\
&= \delta_1 \tau_2^2 \alpha \frac{\kappa^2 - \omega\beta}{a\left(\kappa\tau_0^2\tau_2^2 + \kappa\omega\tau_2^2 + \tau_2^2\kappa^2 + \omega\tau_1^2\tau_2^2 + 2\alpha\omega\tau_2^2 + \kappa\tau_1^2\tau_2^2 + 2\kappa\alpha\tau_2^2 + \tau_0^2\omega\tau_2^2 + \alpha\kappa^2 + \kappa\omega\alpha\right)} \\
\tau_0 &= f^0\left(\tau_2, \tau_1, \tau_0\right) = \frac{\delta_0\left(\sigma_q s + 1\right)\left(c_1 + b_1\right) b_y + s\left(1 - b_y\right)\left(c_1\sigma_{yq} + b_1\sigma_q\right)}{a\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right)}
\end{aligned}
$$

We show the existence in three steps.

**Lemma 1** *Let us fix $\tau_0 = \bar{\tau}_0$ at any arbitrary level. The system $\tau_2 = f^2\left(\tau_2, \tau_1, \bar{\tau}_0\right), \tau_1 = f^1\left(\tau_2, \tau_1, \bar{\tau}_0\right)$ will have at least one fix point, $(\tau_1^*, \tau_2^*)$. Additionally, $\tau_2^{\min} \leq \tau_2^* < \delta_2\frac{1}{a}\alpha$ where $\tau_2^{\min}$ is the single root of $\delta_2\frac{1}{a}\alpha\frac{\kappa\omega+\kappa^2}{\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^2+\omega\tau_2^2} = \tau_2$ and $\frac{\left(\kappa^2-\omega\beta\right)}{a2\kappa} < (>)\tau_1^* \leq (\geq)0$ if and only if $\kappa^2 < (\geq)\omega\beta$. Furthermore, let $\tau_1^*(\tau_0)$ and $\tau_2^*(\tau_0)$ are given as the fixed points corresponding to $\bar{\tau}_0 = \tau_0$ with the smallest absolute value. Then these functions will be continuous.*

**Proof.** Notice first, that $\tau_1 = f^1\left(\tau_2, \tau_1, \bar{\tau}_0\right)$ determines a third degree polynom in $\tau_1$, which is monotone increasing so it gives a single root for every $\tau_0$ and $\tau_2$. Similarly,$\tau_2 = f^2\left(\tau_2, \tau_1, \bar{\tau}_0\right)$ also determines a monotone increasing third degree polynom in $\tau_2$ which gives a single unique root for every $\tau_0$ and $\tau_1$. It is also apparent that a marginal change in $\tau_0$ or $\tau_2$ in the first polynom or a marginal change in $\tau_0$ or $\tau_1$ in the second polynom will cause only a marginal change in the roots. This gives the continuity of $\tau_1^*(\tau_0)$ and $\tau_2^*(\tau_0)$.

For the existence, note that from the root of the polynom $\tau_1 = f^1\left(\tau_2, \tau_1, \bar{\tau}_0\right)$, $\tau_1^2\left(\tau_2^2\right)$ is a well defined continuous function. Therefore, the equilibrium is given by the fixed point of $\tau_2 = \frac{1}{a}$

$\alpha\delta_2 g^2(\tau_2) = f^2\left(\tau_2, \tau_1^2\left(\tau_2^2\right), \bar{\tau}_0\right)$ where $g(\cdot)$ continuously maps $\tau_2$ to the unite interval. As $\lim_{\tau_2\to 0}\frac{1}{a}$ $\alpha\delta_2 g(\tau_2) = \delta_2\frac{1}{a}\alpha\frac{\kappa\omega+\kappa^2}{\tau_1^2(0)\kappa+\kappa^2+\tau_1^2(0)\omega+\tau_0^2\kappa+\alpha\kappa+\tau_0^2\omega+\alpha\omega+2\kappa\omega} > 0$, where $\tau_1^2(0)$ is finite and $0 < \frac{1}{a}\alpha\delta_2 g(\tau_2) \leq \delta_2\frac{1}{a}\alpha$, there has to be a fixed point. The rest of the lemma comes from simple observation. ■

**Lemma 2** *The second order condition of the maximization problem in period 0 always holds, so the denominator of* $f^0(\tau_1, \tau_2, \tau_0)$

$$a\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right) > 0$$

**Proof.** Note that the matrix

$$Q = \left(\left(\begin{array}{cc} 0 & 0 \\ 0 & s \end{array}\right) + \left(\begin{array}{cc} \sigma_y & \sigma_{yq} \\ \sigma_{yq} & \sigma_q \end{array}\right)^{-1}\right)^{-1}$$

is positive definite as $s > 0$. Consequently

$$0 < xQx^T$$

for all $x$.The lemma comes from the choice of

$$x = \left(\left(\begin{array}{cc} -ac_1d_1 & (-ab_1d_1 + sx_i) \end{array}\right) - 2\left(\begin{array}{cc} \mu_y & \mu_q \end{array}\right)\left(\begin{array}{cc} 0 & 0 \\ 0 & \frac{1}{2}s \end{array}\right)\right)$$

as then

$$0 < xQx^T = \left(\begin{array}{c} \frac{a^2\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right)}{\sigma_q s+1}d_1^2 - \\ -\frac{2ac_1\sigma_{yq}sx_i - 2\sigma_q ab_1\mu_q s - 2ac_1\sigma_{yq}\mu_q s + 2\sigma_q ab_1 sx_i}{\sigma_q s+1}d_1 \\ +\frac{s^2\sigma_q\left(x_i-\mu_q\right)^2}{\sigma_q s+1} \end{array}\right),$$

which is possible for all $d_1$ only if

$$a\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right) > 0.$$

■

**Proposition 6** *There is always at least one equilibrium.*

**Proof.** From Lemma 1, we have to show that the expression $\tau_0 = g^0(\tau_0) = f^0(\tau_2^*(\tau_0), \tau_1^*(\tau_0), \tau_0)$ has at least one fix point. We proceed in 4 steps.

1. Note, that $g^0(\tau_0) = g^0(-\tau_0)$ for all $\tau_0$. It is so, because $\tau_0$ enters as $\tau_0^2$ to all building-blocks of $g^0(\tau_0)$.
2. We show that $\lim_{\tau_0\to\infty} g^0(\tau_0) = 0$. Let us check the building-blocks separately. As $\tau_0 \to 0$, $\tau_2^*$ goes to a constant, $\tau_1^*$ goes to 0 by the order of $\frac{1}{\tau_0^2}$, hence $c_1, \sigma_y, s$ goes to constants, $\sigma_{yq}$ and $b_y$ goes to 0 by the order of $\frac{1}{\tau_0^2}$, $\sigma_q$ goes to infinity by the order of $\tau_0^4$ and $b_1$ goes to 0 by

the order of $\frac{1}{\tau_0^2}$ . So the nominator of $g^0(\tau_0)$, $(\sigma_q s+1)(c_1+b_1)b_y+s(1-b_y)(c_1\sigma_{yq}+b_1\sigma_q)$, goes to infinity by the order of $\tau_0^2$ from the speed of convergence of the term $\sigma_q s(c_1 b_y+b_1)$. The denominator,$a\left(c_1^2\sigma_y+2c_1\sigma_{yq}b_1+c_1^2 s\sigma_y\sigma_q-c_1^2 s\sigma_{yq}^2+\sigma_q b_1^2\right)$ also goes to infinity but by the order of $\tau_0^4$ given by the term of $c_1^2 s\sigma_y\sigma_q$. Hence, $\lim_{\tau_0\to\infty} g^0(\tau_0)=0$.

3. The function $g^0(\tau_0)$ is continuous. It comes by the positivity of the denominator, which holds because of Lemma 2, and the continuity of all building-blocks.

4. Hence, $g^0(\tau_0)$ will cross the 45° line necessarily, because it is symmetric, continuous and goes to 0 as $\tau_0$ increases without bound. Therefore, there will be a fixed point with $\tau_0^*\geq(<)\,0$ if $g^0(0)\geq(<)\,0$.

■

## A.4 Proof of Proposition 5

**Proof.** The second half of the statement implies the first half as both the aggregate holdings and the volume are continuous functions of $\omega$. For the second half of the statement, it is sufficient to show that $\lim_{\omega\to\infty}\left|\frac{\tau_0^n}{\delta_0}-\frac{\tau_1^n}{\delta_1}\right| = \lim_{\omega\to\infty}\left|\frac{\tau_0^n}{\delta_0}\right| = \lim_{\omega\to\infty}\left|\frac{\tau_1^n}{\delta_1}\right| \to 0$ while $\left|\frac{\tau_0}{\delta_0}-\frac{\tau_1}{\delta_1}\right|\to C_1, \left|\frac{\tau_1}{\delta_1}\right|\to C_2$ and $\left|\frac{\tau_0}{\delta_0}\right|\to C_3$ , where $C_1, C_2$ and $C_3$ are non zero constants. In the no-announcement case, the equilibrium is characterized by the following equations:

$$
\begin{aligned}
\tau_2^n &= f^2\left(\tau_2^n,\tau_1^n,\tau_0^n\right) = \\
&= \frac{\delta_2\frac{1}{a}\alpha\left(\tau_0^{n2}\omega+\tau_1^{n2}\omega+\kappa\omega+\tau_0^{n2}\kappa+\tau_1^2\kappa+\kappa^2\right)}{\tau_0^{n2}\kappa+\tau_0^{n2}\tau_2^2+\tau_0^{n2}\omega+\alpha\tau_0^{n2}+\alpha\tau_1^{n2}+\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^{n2}+\omega\tau_2^{n2}+\tau_1^{n2}\tau_2^{n2}+\tau_1^{n2}\omega+\tau_1^{n2}\kappa} \\
\tau_1^n &= f^1\left(\tau_2^n,\tau_1^n,\tau_0^n\right) = \\
&= \delta_1\tau_2^2\alpha\frac{\kappa^2}{a\left(\kappa\tau_0^{n2}\tau_2^2+\kappa\omega\tau_2^2+\tau_2^2\kappa^2+\omega\tau_1^{n2}\tau_2^2+2\alpha\omega\tau_2^2+\kappa\tau_1^{n2}\tau_2^2+2\kappa\alpha\tau_2^2+\tau_0^{n2}\omega\tau_2^{n2}+\alpha\kappa^2+\kappa\omega\alpha\right)} \\
\tau_0^n &= f^0\left(\tau_2^n,\tau_1^n,\tau_0^n\right) = \delta_0\frac{b_y^n+\sigma_q^n s^n}{b_1^n a\sigma_q^n}.
\end{aligned}
$$

It is apparent, that for any $\tau_1^n,\tau_0^n$, $\lim_{\omega\to\infty}\tau_2^n$ is a finite, positive constant, and for any finite, positive $\tau_2^n$ and any $\tau_0^n$ $\lim_{\omega\to\infty}\tau_1^n=0$. Consequently, $\lim_{\omega\to 0}\sigma_q^n=\lim_{\tau_1\to 0}\sigma_q^n=\infty$. Hence,

$$
\lim_{\omega\to\infty}\delta_0\frac{b_y^n+\sigma_q^n s^n}{b_1^n a\sigma_q^n} = \lim_{\omega\to\infty}\frac{s^n}{b_1^n} = \lim_{\omega\to\infty}\frac{\frac{b_s^n}{\left(\sigma_s^n+\frac{1}{\tau_2^2}\right)}}{b_2^n\frac{(b_s^n+e_s^n)}{b_s^n}+\frac{\bar{f}}{b_s}} = 0
$$

, which holds because $\lim_{\omega\to\infty}b_s^n=0$, but $\lim_{\omega\to\infty}b_2^n$, $\lim_{\omega\to\infty}\sigma_s^n$ and $\lim_{\omega\to\infty}\frac{(b_s+e_s)}{b_s}=\lim_{\omega\to\infty}\frac{\tau_1^2\kappa^2+\alpha\kappa^2}{\alpha\kappa^2}=1$ are non zero constants.

In the announcement case, our equilibrium determining equations go to the following ones as

$\omega \to \infty:$

$$
\begin{aligned}
\tau_2 &= \delta_2 \frac{1}{a} \alpha \frac{\left(\kappa + \tau_0^2 + \tau_1^2\right)}{\left(\tau_0^2 + 2\kappa + \tau_2^2 + \tau_1^2 + \alpha\right)} \\
\tau_1 &= -\delta_1 \tau_2^2 \alpha \frac{\beta}{a\left(\tau_1^2 \tau_2^2 + \kappa \tau_2^2 + \tau_0^2 \tau_2^2 + 2\alpha \tau_2^2 + \kappa \alpha\right)} \\
\tau_0 &= \frac{\delta_0 \left(\sigma_q s + 1\right)\left(c_1 + b_1\right) b_y + s\left(1 - b_y\right)\left(c_1 \sigma_{yq} + b_1 \sigma_q\right)}{a\left(c_1^2 \sigma_y + c_1^2 s \sigma_y \sigma_q - c_1^2 s \sigma_{yq}^2 + \sigma_q b_1^2 + 2 c_1 \sigma_{yq} b_1\right)},
\end{aligned}
$$

where the building-blocks of the last equations are all of the corresponding limiting functions. By the observation of expressions for the building-blocks, it is apparent that all of them are going to finite, non-zero constants as $\omega \to \infty$. Hence, just by the same reasoning as in the existence theorem, there must be at least one equilibrium where all $\tau_2, \tau_1, \tau_0$ will be finite and non-zero. If $\lim_{\omega \to \infty} \frac{\tau_1}{\delta_1}$ and $\lim_{\omega \to \infty} \frac{\tau_0}{\delta_1}$ happened to be equal with certain combinations of the parameters, small perturbation on the parameters (for example perturbing $\delta_0$) would unambiguously make $\lim_{\omega \to \infty} \left| \frac{\tau_0}{\delta_0} - \frac{\tau_1}{\delta_1} \right| > 0$. ■