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Estimating yield curves from swap, BUBOR and FRA data

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Estimating yield curves from swap, BUBOR and FRA data

(Hozamgörbebecslés kamatswap, BUBOR és FRA adatokból)

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Abstract

In this paper we estimate yield curves from Hungarian interest rate swap and money market data. Following general practice, we experiment with several models—differing in the functional form and objective function—and chose the model which performs best according to standard evaluation criteria. We find that the methods perform equally well in terms of residuals and out-of-sample fit; however, the smoothing spline method stands out when we consider the ability to fit the short end of the maturity spectrum, stability of estimation and plausibility of the estimated curves.

JEL Classification: E43, G12.

Keywords: yield curve, interest rate swaps.

Összefoglalás

Ebben a tanulmányban hozamgörbét becsültünk BUBOR és FRA hozamok, valamint forint kamatswap jegyzések felhasználásával. Az általános gyakorlatnak megfelelően a becslést – mind az alkalmazott függvényforma, mind a célfüggvény tekintetében – többféle módszerrel is elvégeztük, és a kapott eredmények összehasonlításával választottuk ki a mindennapi használatra legalkalmasabbnak tűnő modellt. Bár az illeszkedés pontossága és a mintán kívüli hibák az esetek többségében nagyon hasonlóak voltak, a rövid horizontú illeszkedés, a kapott görbék értelmezhetősege és a stabilitási tulajdonságok alapján a simító spline módszer bizonyult a legelfogadhatóbbnak.

1 Introduction

When assessing the effects of monetary policy decisions the expectations of economic agents play a major role. In an inflation targeting regime the central bank sets interest rates to influence expected returns and financing costs of investments, which in turn determine inflation through various real-economy channels.[1]

However, these real effects will occur only if the decision takes the agents by surprise. Since investment and financing decision are based on expected future returns, these decisions can only be altered if the expectations are altered. A proper assessment of what the agents' expectations are is therefore of paramount importance in making policy decisions.

Unfortunately, expectations are not directly observable, therefore we have to estimate them using observable data. One possible way of doing this is the estimation of yield curves, where we use observed prices and cashflows of financial assets to estimate the discount factor applied in pricing these assets, and convert the discount factor into forward rates. The expectation hypothesis then implies that the forward rates can be interpreted as expectation of future returns.

The result of course depends on the type of assets we use in the estimation. The most widespread practice in central banks is to estimate yield curves from government bond prices, as in each country these are considered to be riskless assets, and therefore their prices are unlikely to be contaminated by individual risk considerations. Details about the estimation of the government bond yield curve currently employed by the Magyar Nemzeti Bank (MNB, the central bank of Hungary) are discussed in Gyomai & Varsányi (2002); see also Csajbók (1998).

Besides considerations of riskiness, liquidity of the assets is also a crucial issue. Similarly, reliable results can only be achieved if the data covers a sufficiently large maturity spectrum. The Hungarian government bond market has limitations in both respects. Firstly, the shortest maturity available is usually around three months, making the short end of the estimated yield curve a mere extrapolation depending on the functional form assumed. Secondly, mispricing of some less frequently traded maturities often leads to implausible estimations. It might therefore be worth examining whether the use of alternative sources of data can help to circumvent these problems.

As Balogh *et al.* (2007) shows, the forint interest rate swap market is about the same size as the market of government bonds, and the bid-ask spreads and average transaction size indicate that it is probably more liquid. This means that—although, due to the hedging activity of market makers, the two markets contain the same information—the swap market might react to this information earlier. Since the reference rate of the swaps is the Budapest Interbank Offer Rate (BUBOR), using BUBOR rates to supplement the swap data might give more reasonable estimates of the short and of the yield curve, as BUBOR rates are available for maturities as short as two weeks.

In this paper the methods and results of estimating yield curves from forint interest rate swap and money market data are presented. A brief theoretical overview is given in Section 2, and the data is described in Section 3. The results of various estimation methods are presented and discussed in Section 4. Section 5 concludes.

[1] A short description of the monetary transmission mechanism in Hungary can be found in Vonnák (2007).

2 Theoretical background

BASIC DEFINITIONS

If a bond with maturity T has coupon dates $t_1, t_2, \ldots, t_n = T$, and the coupon payments are $c_1, c_2, \ldots, c_n$, then the bond's price in $t < t_1$ is given by

$$P_t = \delta(t, t_n - t) + \sum_{i=1}^{n} \delta(t, t_i - t) c_i. \tag{1}$$

The function $\delta(t, h)$ is called the discount factor, which gives the value at t of a unit payment that is due at $t + h$. The *expectation hypothesis* asserts that—using continuous compounding—the discount factor can be written in the form

$$\delta(t, h) = e^{-\int_0^h f(t,s)ds}, \tag{2}$$

where $f(t, s)$ denotes the expectation, at time t, of the short (instantaneous) interest rate that will prevail at time $t + s$; therefore $f(t, s)$, as a function of s, is called the *instantaneous forward curve* at time t.[2] Obviously, the discount factor can be calculated from the forward curve using

$$f(t, h) = -\frac{\partial}{\partial h} \log \delta(t, h).$$

ESTIMATION OF THE YIELD CURVE

Yield curves can be estimated from observed bond prices and known coupon payments by minimizing the difference between the observed prices and the prices calculated from Equation 1. To be able to use Equation 1, we usually assume that the forward curve is parameterized, i.e., $f(t, h) = f(\pi_t, h)$, where π_t is a vector of parameters. Time dependence of the yield curve is therefore represented by the time dependence of the parameters, which are reestimated in every period.

Functional forms

In the empirical literature the most widely used function forms are the following.[3]

Nelson-Siegel

$$f(h) = \beta_0 + \beta_1 e^{-h/\tau_1} + \beta_2 \frac{h}{\tau_1} e^{-h/\tau_1}. \tag{3}$$

This function is the solution of a second order linear differential equation with one real root. The intuition given in Nelson & Siegel (1987) is that if the evolution of the short rates can be described by such a differential equation, then expectations should be given by the solution of this equation.

The three components of the function are usually interpreted as long term (β_0), short term $\left(\beta_1 e^{-h/\tau_1}\right)$ and medium term $\left(\beta_2 \frac{h}{\tau_1} e^{-h/\tau_1}\right)$ factors, since the second term vanishes at $h = \infty$ and the third term vanishes at $h = 0, \infty$.

The Nelson-Siegel form can describe increasing and decreasing yield curves as well as curves with one local maximum or minimum (hump-shaped or inverted hump-shaped curves).

[2] In this paper we use the terms yield curve, forward curve and instantaneous forward curve as synonyms.

[3] For the sake of simplicity from now on we will write $f(h)$ instead of $f(t, h)$ and $f(\pi, h)$ instead of $f(\pi_t, h)$. Methods that explicitly model the time-path of the parameters are discussed in Diebold & Li (2005) and Diebold *et al.* (2005).

Svensson

$$f(h) = \beta_0 + \beta_1 e^{-h/\tau_1} + \beta_2 \frac{h}{\tau_1} e^{-h/\tau_1} + \beta_3 \frac{h}{\tau_2} e^{-h/\tau_2}. \tag{4}$$

The difference between the Nelson-Siegel and the Svensson forms is that the latter introduces an extra medium term factor to allow for greater flexibility, see Svensson (1994). Greater flexibility in this case is the ability to produce curves with two extrema, one maximum and one minimum or vice versa. This is the form most commonly applied by central banks, the MNB's current government bond yield curve also has this form.

Spline

By definition, a spline function is a piecewise polynomial, smooth function.[4] A spline is defined by the end points of the intervals on which the function is polynomial (the so called *knots*), the degree of the polynomials, and the order up to which the function is differentiable at the knots. In the most widespread case—and this is the case that we will exclusively deal with—the polynomials are of order three and smoothness means twice differentiability.

It is easy to see that every such function is a linear combination of certain so called *basis functions*:

$$f(h) = \sum_{i=1}^{m} \gamma_i B_i(h, \varkappa), \tag{5}$$

where the basis functions depend only on $\varkappa$, the vector of knots. If the knots are fixed independent of the data, following some rule-of-thumb type reasoning—which is standard practice—, then the parameters to be estimated are the γ_i-s. With third degree polynomials and second order smoothness the number of parameters is the number of knots minus two.

Since the knots can be chosen anywhere within the maturity spectrum, splines are extremely flexible in shape. This flexibility often leads to implausible forward curves, i.e., curves that exhibit undesirably high volatility. To avoid such results, the objective function is usually augmented with an extra term that penalizes excess curvature. In such case the estimation is referred to as fitting a *smoothing spline*.

Objective function

As we have already mentioned, the parameters of the yield curve are usually estimated by minimizing the pricing errors. Let's suppose we observe the prices of N bonds, and let the vector of these prices be $P \in \mathbb{R}^{N\times 1}$. Furthermore, let h be the vector that consists of all the points in time when any of the bonds has a payoff (including maturities): $h \in \mathbb{R}^{K\times 1}$. Let row i, column j entry in the matrix $C \in \mathbb{R}^{N\times K}$ be c_i^j if bond i has a payoff c_i^j at time h_j, including principal at maturity.

Given a parameter vector π, the theoretical prices are given by $C\,\delta(\pi, h)$, where $\delta(\pi, h)$ is the vector whose entries are $\delta(\pi, h_j)$.[5] The least squares objective function now can be written as

$$\rho(\pi) = (P - C\,\delta(\pi, h))'(P - C\,\delta(\pi, h)). \tag{6}$$

This must be minimized numerically since the discount factor is usually a nonlinear function of the parameters.

[4] For a discussion of spline functions see Fisher *et al.* (1995).

[5] Using the definition of the forward curves and Equation (2), it is possible to analytically derive the formulas of the discount factors in all three cases.

By using least squares we implicitly assume that the residuals satisfy the properties of homoscedasticity and no autocorrelation. In terms of the problem at hand, these properties mean that all the bonds carry the same amount of information and that there is no significant term premium in the data.

The augmented objective function of a smoothing spline model can be defined in two ways:

$$\begin{aligned}\rho(\pi) &= (P - C\,\delta(\pi,h))'(P - C\,\delta(\pi,h)) + \lambda \int_0^H \left[f''(\pi,s)\right]^2 ds,\\ \rho(\pi) &= (P - C\,\delta(\pi,h))'(P - C\,\delta(\pi,h)) + \int_0^H \lambda(s)\left[f''(\pi,s)\right]^2 ds,\end{aligned} \tag{7}$$

where $H = \max(h)$ is the longest maturity in the data. In both cases curvature is measured by the integral of the square of the second derivative of the spline. In the first case, volatility is penalized uniformly in the whole maturity spectrum, while in the second case it is possible—by a suitably choice of the function $\lambda(s)$—that the curvature is penalized more heavily in certain sections (usually at the long end) of the maturity spectrum. An example of the first approach is Fisher *et al.* (1995), while the second approach is followed in Waggoner (1997), Anderson & Sleath (2001) and Gyomai & Varsányi (2002).

Instead of minimizing pricing errors, the objective function may be written up in terms of yields to maturity, as is done in Svensson (1994). Another possibility is to transform the data into forward rates and fitting the models to these rates; examples are Diebold & Li (2005) and Diebold *et al.* (2005). Forward rates are calculated by the method of Fama & Bliss (1987), by sequentially pricing longer maturity bonds using the assumption that forward rates are constant between successive maturities. The associated discount factor prices every bond without error, so by switching to Fama-Bliss rates no information is lost. If Fama-Bliss forward rates at h are denoted by $\phi(h)$ then the objective function becomes

$$\rho(\pi) = (\phi(h) - f(\pi,h))'(\phi(h) - f(\pi,h)). \tag{8}$$

This now depends directly on the forward curve, which is especially useful in case of the spline models, where—according to (5)—the forward curve is a linear function of the parameters, so they can be estimated by OLS without having to use numerical methods. The roughness penalty terms can be added exactly as above.

We now turn to the description of the data; further technical details about the estimation can be found in Section A of the Appendix. A detailed discussion of the topics briefly presented here can be found in Anderson *et al.* (1996).

3 The data

In the estimation we used three types of data: BUBOR yields,[6] *forward rate agreement* (FRA) yields and interest rate swap yields. The joint treatment of these data is justified by the fact that the reference yield of the FRA-s and swaps is the BUBOR. The shortest maturity was the two week BUBOR, the longest the twenty year swap. The sample period consisted of 313 days, from 3rd July 2006 until 31st October 2007.

SWAP YIELDS

A thorough treatment of swap contracts, the forint interest rate swap market and the data can be found in Balogh *et al.* (2007), so here we only list those the properties that are relevant for estimation. Swaps are quoted at par yields: this is the rate of the fixed leg of the contract, calculated to make the net present value of the contract equal zero. A short calculation shows that this implies that if the discount factor is calculated from the expected values of the floating leg, then the present value of a hypothetical bond that has coupon rate equal to the fixed leg equals the principal.

The maturities of the contracts are from 1 to 10, 12, 15 and 20 years. We used quotes of the London based interdealer broker ICAP, which was accessed through Reuters. The Reuters database consists of only mid-quote rates for the new issues.

Figure 1
Swap yields

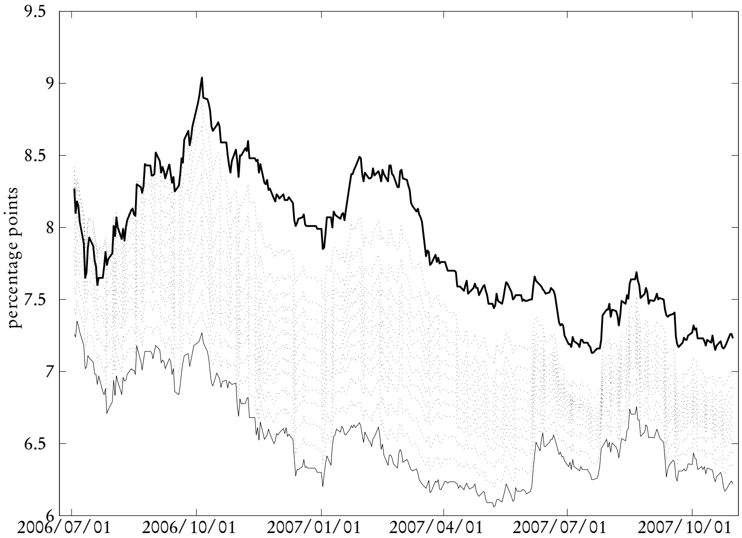


Figure 1 gives a broad picture of the data. The thick solid line shows one year yields, the thin solid line shows 20 year yields, other maturities are shown in dotted lines. On the face of it, the figure indicates that swap yields were decreasing with maturity and most of the variation came from level shifts.

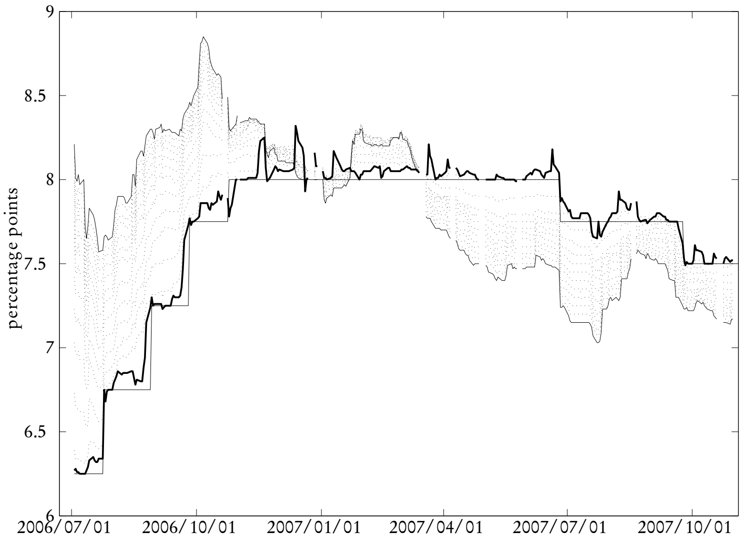
BUBOR YIELDS

We used BUBOR rates for maturities of two weeks and from one to twelve months. Although overnight and one week rates are also available, these are likely to be driven by short term liquidity concerns, and therefore we decided to disregard them.

[6] BUBOR stands for Budapest Interbank Offer Rate.

Figure 2 shows the data together with the two week MNB base rate. It is worth noting that in the first half of the sample period, roughly before the end of 2006, BUBOR rates clearly indicated expectations of increasing returns, which can not be seen in swap yields. It is also evident that changes of the slope contribute significantly to the overall variation, in conjunction with fact that the short end of the yield curve may exhibit higher volatility than the long end.

Figure 2
BUBOR rates and MNB base rate



FRA YIELDS

A forward rate agreement is similar to a swap with only one exchange of interest payments. The only difference is that in traditional swaps the floating leg is determined *one period before* the actual exchange takes place, while the reference rate of an FRA is the floating rate prevailing *at the time* of the exchange. An FRA contract is classified by the date of exchange relative to the contract date and the horizon of the reference floating rate. That is, in case of a 1×4 FRA, the exchange takes place one month from now and the reference rate is the three month BUBOR prevailing at that time. Figure 3 depicts the twelve FRA yields that were used in the estimation, from 1×4 to 12×15.

CONSISTENCY OF BUBOR AND FRA DATA

As it was mentioned before, the possibility of incorporating all three kinds of data into a single estimation is grounded in the fact that both the swap and FRA contracts specify the BUBOR as their reference rate. Nevertheless, we are dealing we three distinct markets, therefore an examination of the consistency of the data is necessary.

We will focus on the consistency of the BUBOR and FRA yields, since these yields carry information about the same maturity spectrum, namely the short end of the yield curve. More precisely, we compare a BUBOR rate with a rate that is calculated by coupling a shorter BUBOR rate with an FRA of appropriate horizon, e.g., a seven month BUBOR with a rate calculated from four month BUBOR and 4×7 FRA. As can easily be seen, for an i month BUBOR this can be done in only one way if $i = 4, 5, 6$, in two ways is $i = 7, 8, 9$ and in three ways if $i = 10, 11, 12$ (in the example above, the seven month "BUBOR" can also be constructed from one month BUBOR, 1×4 FRA and 4×7 FRA).

Figure 3
FRA yields

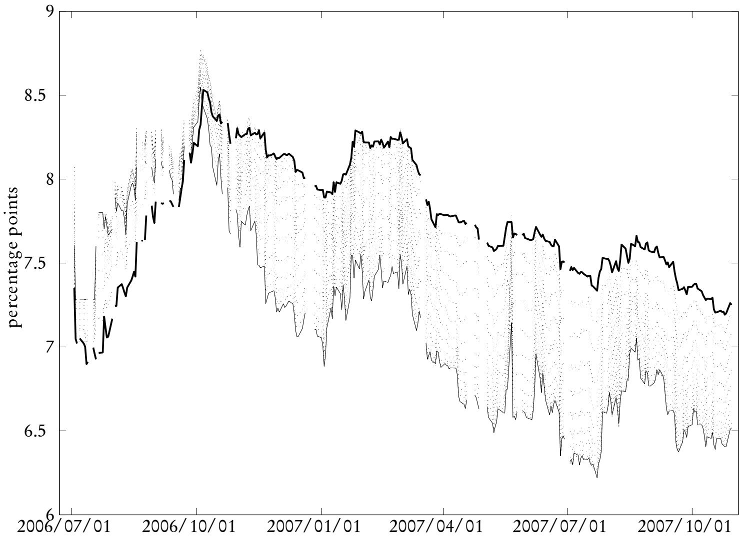
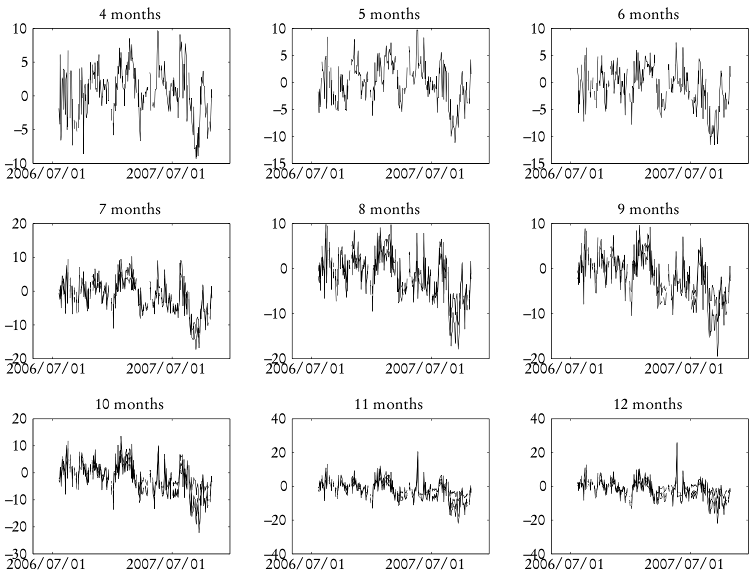


Figure 4 shows the differences of the calculated and actual BUBOR rates; the plots in the first row contain one series, plots in the second row contain two series, and plots in the last row contain three series. Table 1 shows the percentiles of these differences, pooled together in time and across the maturities used in the calculation.

We see that the errors are broadly within the range of ±10 basis points, and the time paths show no sign of systematic departure from zero. Taking into account that we only have mid-quote FRA yields and that the BUBOR is an offer rate by definition, such discrepancies seem to be acceptable, and there is no evidence against the use of both types of data in the estimation.

Figure 4
Consistency of BUBOR and FRA data (basis points)



FORECASTING PROPERTIES OF FRA DATA

We complete the description of the data by examining the forecasting properties of FRA yields. An $i \times (i+3)$ FRA is the i-months ahead expectation of the three month BUBOR, so we can calculate forecast errors from

Table 1

Consistency of BUBOR and FRA data, percentiles of differences

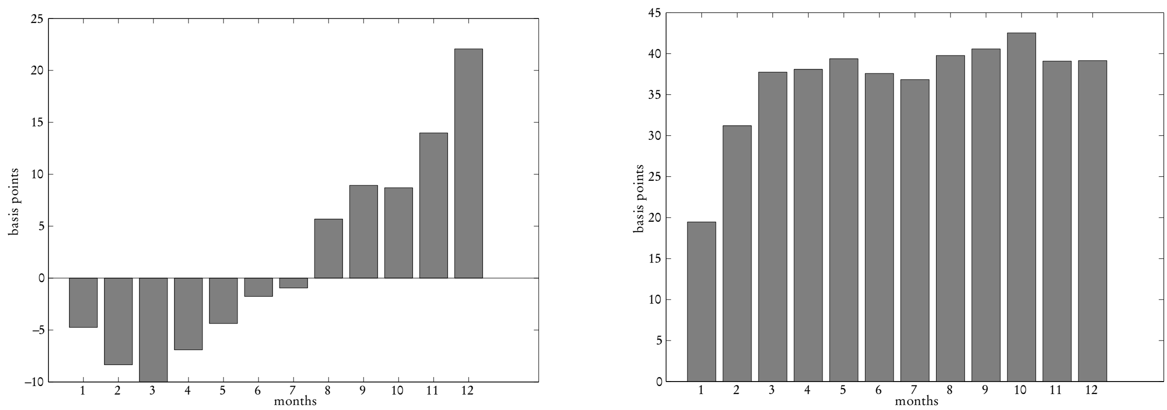
	0.025	0.25	0.5	0.75	0.975
4m	−7.60	−1.82	0.91	2.85	7.67
5m	−8.32	−2.66	0.06	2.44	6.76
6m	−9.50	−3.26	−0.75	1.67	5.05
7m	−12.49	−4.57	−1.04	1.82	7.11
8m	−12.09	−4.46	−0.96	1.89	6.57
9m	−11.55	−4.88	−1.27	1.54	6.39
10m	−13.52	−5.33	−1.71	1.36	7.46
11m	−12.82	−5.26	−1.75	1.15	7.34
12m	−11.78	−4.87	−1.64	1.22	6.85

one to twelve months ahead. The means of these errors are shown in Figure 5(a) and the root mean squared errors are in Figure 5(b).

We are primarily interested in whether FRA rates can be viewed as unbiased forecasts of future BUBOR rates, i.e., whether the mean of the forecast errors is significantly different from zero. Although a proper assessment of this question would require a more precise handling of the overlapping forecast periods, at first glance the figures do not seem to firmly reject the hypothesis of zero mean forecast errors.

Figure 5

Forecast errors of FRA yields



(a) mean (b) RMSE

4 Estimation results

We now turn to the presentation and discussion of the estimation results end estimated yield curves. Since we have no *a priori* criteria to decide upon a particular functional form or objective function, we estimate several specifications and chose the one that proves to be the most appropriate for day-to-day application.

We estimated six curves on each day in the sample period. We used three functional forms: the Svensson function, smoothing spline with fixed knots and unsmoothed spline with variable knots.[7] We fitted all three functional forms to both prices and Fama-Bliss forward rates.

Our motivation for experimenting with variable knot splines was the desire to get rid of the need for an *ad hoc* smoothing function. By decreasing the number of knots while allowing them to adjust to the data at hand, it might be possible to eliminate the excess curvature that is usually caused by overfitting the observations that are close to the predefined knots. Although some references, e.g., Fernandez-Rodriguez (2006), claim to have been successful in estimating knots from data, we found only limited support for this approach.

Table 2

The estimated models

	Functional form	Objective function	Knots	Smoothing
SV	Svensson	Pricing errors	-	-
SV_FB	Svensson	Yield errors	-	-
SPL	Spline	Pricing errors	fixed	yes
SPL_FB	Spline	Yield errors	fixed	yes
SPLV	Spline	Pricing errors	variable	no
SPLV_FB	Spline	Yield errors	variable	no

Table 2 gives a quick overview of the six models, together with abbreviations that will henceforth be used to refer to the models.

The results will be evaluated according to several criteria:

- residuals and out-of-sample errors indicate the ability to fit the observed yields and the robustness of the results to the maturities used in the estimation,
- stability tests also check for robustness, but this time the sensitivity of the results to small perturbations of the input yields is assessed,
- by looking at forecast errors we examine whether forward rates calculated from the estimated curves can be viewed as unbiased forecasts of future BUBOR rates,
- finally, we enquire into the possible economic interpretations of changes in the estimated curves—especially at the short end—, and whether such changes can be attributed to developments in the economic environment. As we do not explicitly model the dynamics of the yield curve, this enquiry will be conducted only by "visual inspection" of the graphs of short and long rates.

RESIDUALS

Table 3 shows the square roots of the (simple) averages of the squared residuals, where the average has been taken over all days in the sample period and over all maturities of the specific type of input yield.

[7] After preliminary calculations we found that the Nelson-Siegel model had very poor in sample fit.

Fitting to Fama-Bliss yields provides considerably larger swap and FRA residuals and marginally smaller BUBOR residuals than fitting to prices. We interpret this as the inability of the functional forms to replicate the shape of the piecewise constant Fama-Bliss forward curve.

Table 3

Pricing errors (in basis points)

	SV	SV_FB	SPL	SPL_FB	SPLV	SPLV_FB
Swap	9.13	22.01	7.75	15.84	8.97	15.36
BUBOR	14.14	12.79	10.50	9.56	10.03	9.73
FRA	12.74	18.85	10.71	18.93	9.87	18.80

Tables 11, 12 and 13 in Appendix B show the residuals by maturities. We see that the larger swap and FRA errors of the yield-fitting methods are the results of larger errors at the longer maturities; indeed, the SPL_FB and SPLV_FB models fit the short FRA data much better than their price-fitting versions.

Considering only the price-fitting models, there seems to be no substantial difference between the functional forms in terms of the swap residuals (although the almost perfect fit of the SPLV model at the longest maturity worth mentioning), while the SV model fits the very short BUBOR rates much worse, and the SPLV model fits the long FRA rates much better than the other two specifications.

All in all, residuals provide no clear clue as to which method has the best performance, although the price-fitting estimations seem to have a better overall fit than yield-fitting methods.

OUT-OF-SAMPLE ERRORS

Out-of-sample errors were calculated for each type of yield and each maturity[8] by leaving out one data point, estimating the model, and calculating the pricing error for the yield not used in the estimation. Anderson & Sleath (2001) calls this *leave-out-one cross validation*; a more thorough exercise would be to leave out a larger set of data from the estimation, as is done in Bliss (1996), who uses every second bond in the estimation and calculates out-of-sample error for the other half of the sample. However, with our samples consisting of only thirty eight observations per day, this was not possible, so we opted for the simpler approach.

Comparing Table 4 with Table 3 indicates that average out-of-sample errors are only marginally larger than in-sample fits. This means that if one maturity is left out from the estimation, the results do not change much, so—at least on the average—there is no sign of overfitting.

Table 4

Out-of-sample errors (in basis points)

	SV	SV_FB	SPL	SPL_FB	SPLV	SPLV_FB
Swap	13.84	22.05	9.72	15.77	14.21	15.52
BUBOR	15.65	14.44	11.62	10.46	11.07	10.53
FRA	13.65	20.23	12.63	20.98	11.40	20.81

However, the picture changes when we look at the out-of-sample errors by maturities. As Table 14 shows, in case of the SPLV model, overfitting *is* present for long maturity swaps: the out-of-sample error of the 15 year swap is 30 basis points while the residual is only 3 basis points. A further proof is Figure 11(a), which clearly shows that the forward curves estimated by the SPLV method exhibit unreasonable volatility at the long end.

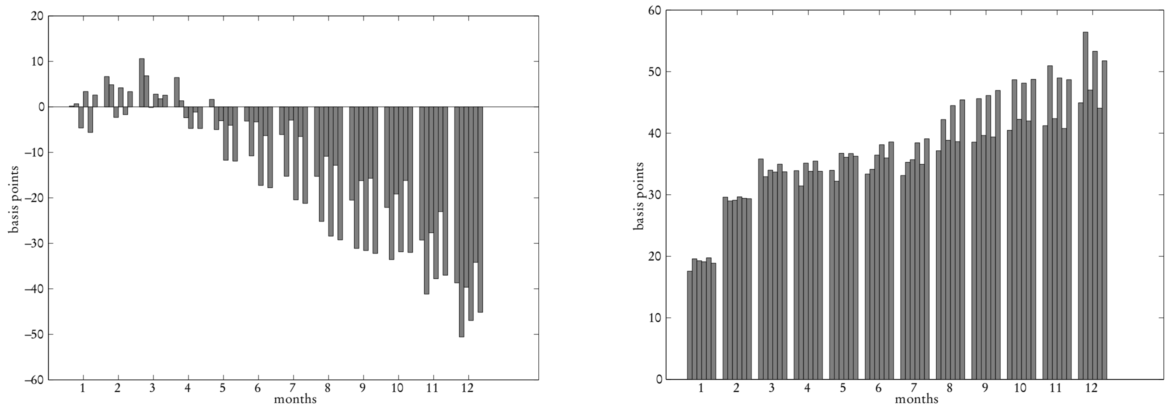
[8] For spline functions the shortest and longest maturities must always be kept in-sample.

Apart from indicating the overfitting of the SPLV model, the behavior of the out-of-sample errors is very similar to that of the residuals: yield-fitting methods are inferior in case of longer maturity swaps and FRA rates, and among the price-fitting methods, the SV method is clearly inferior to the other two in fitting short BUBOR rates.

FORECASTING PROPERTIES

In central banking practice the most important application of the yield curve is to extract expectations of future interest rates. Since it is usually assumed that market expectations are rational, it is a natural requirement that the expectations extracted from the yield curve have the properties of rational expectation. The most important such property is unbiasedness, i.e., forecast errors close to zero.

Figure 6
Forecast errors



(a) mean (b) RMSE

Figure 6 shows the means and the root mean squared errors of the forecasts of the three month BUBOR rate, calculated each day as the integral of the estimated forward curve from $i/12$ to $(i+3)/12$, where i is the forecast horizon in months. For each forecast horizon there are six columns corresponding to the six models; the models are ordered as in Table 2. The results can be compared to those pictured in Figure 5 since the variable to be forecast is the same.

We see that up to four months the mean errors of all models are almost negligible, less than ten basis points in absolute value. From five months the errors begin to increase, but in the best cases—which are the price-fitting methods—they never exceed forty basis points. Considering also the volatility of the errors in Figure 6(b), we can safely draw the conclusion that the mean of the forecast errors is not significantly different from zero.

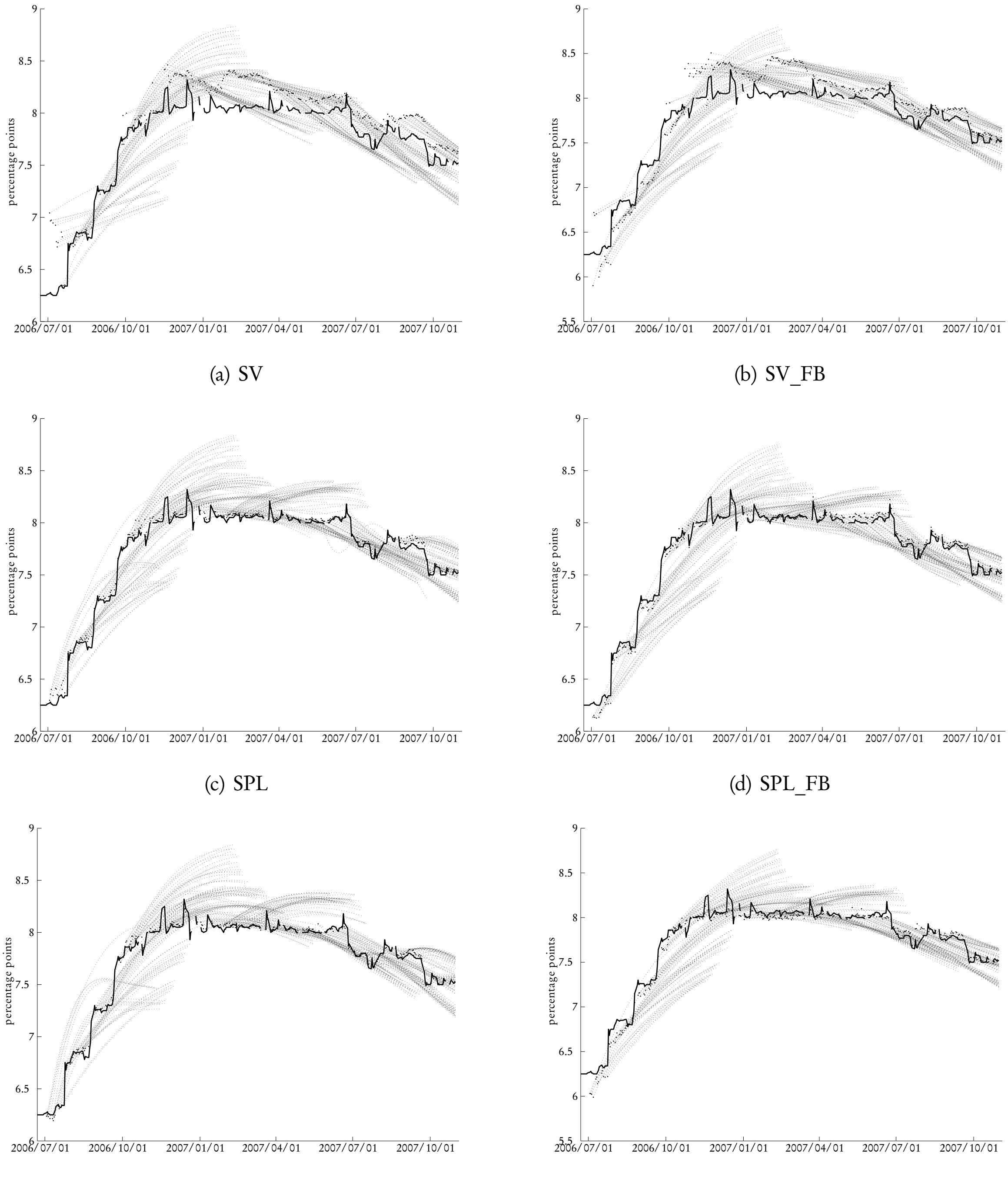
SHORT RATES

As it was mentioned in the introduction, a major benefit of the current estimation exercise—relative to the yield curves estimated from government bond data—is that the very short end of the yield curve is not a result of mere extrapolation: the shortest maturity used is the two week BUBOR rate, while the shortest maturity of the government bonds that are actively traded is usually around three months.

Figure 7 depicts the two-week forward rates for three months ahead calculated from the six models (dotted lines) together with the actual two-week BUBOR rates (solid line). Our aim is to assess the plausibility of the changes in the curves, and—since dynamics is not explicitly handled in the models—we will focus on properties that are deducible by "visual inspection".

The sample period can be divided into three parts (see Figure 2): the first, before October 2006, was characterized by sharply rising MNB base rates; in the second, between October 2006 and June 2007, the base

Figure 7
Short rates



(a) SV (b) SV_FB (c) SPL (d) SPL_FB (e) SPLV (f) SPLV_FB

rate was on hold; in the third, the base rate started to decline slowly. This overall pattern is well reflected by all models. Also, all models indicate that at the end of the tightening cycle expectations of future raises disappeared only gradually, while at the beginning of the easing cycle expectations quickly adjusted to the new environment.

There are two respects in which the models behave differently, and then the difference is only between the two Svensson and the four spline models. First, and this is especially true in the hiking period, the starting points of the curves often deviates significantly from the actual BUBOR rates (the difference can be as large as 75 basis points), which clearly renders interpretation rather difficult.

Second, the Svensson curves often exhibit sudden changes in the slope, which is usually accompanied by a jump of the starting point of the curves in the opposite direction, i.e., when the slope suddenly becomes negative, the starting point jumps up. Two such periods are at the end of the hiking cycle and in (roughly) February and March 2007; in both of these periods the two-week BUBOR rates were relatively flat.

A possible explanation might be fact that, as shown by Figure 2, these are the only periods when the ”middle horizon” BUBOR rates are not between the shortest and the longest maturities. Furthermore, longer swap rates increased during the second period. The strange behavior of the Svensson curves is therefore very likely a sign of the inability of this functional form to separate movements on different maturity sections of the yield curve. This drawback is documented also by Anderson & Sleath (2001) and Gyomai & Varsányi (2002).

Since the spline methods do not show any of the above mentioned anomalies of the Svensson model, plausibility of the short end of the estimated curves favors the usage of the spline functional form.

STABILITY

Since yield curves are routinely estimated and interpreted on a daily basis, it is important to make sure that the noise in the data can not not make its way through to the estimated curves. We therefore examine the stability properties of the estimation methods to assess their ability to filter out irrelevant volatility in the observed data.

Such volatility may stem from two sources. First, yields are reported on a discrete scale, which introduces observation error. Second, lack of liquidity at some—usually longer—maturities may result in quotes that are ”mispriced” relative to the more liquid maturities. As for the first case, we are interested in the size of the change in the estimated curve when the input yields are subject to small random perturbations at all maturities, while in the second case we examine the effect of perturbations to selected long maturities.

More formally—following Anderson & Sleath (2001)—, we will use the standard Euclidean norm to measure the size of the input yields x:

$$||x||_2 = \sqrt{\sum x_i^2}.$$

We will use two norms for the size of the yield curves f:

$$\begin{aligned} ||f||_1 &= \frac{1}{H-h}\int_h^H |f(s)|\,ds, \\ ||f||_\infty &= \max_{h\leq s\leq H} |f(s)|, \end{aligned} \tag{9}$$

where h and H are the shortest and longest maturities. Using these norms, we can define the *condition number* of the estimation method A—which is a mapping from x to f—as

$$\begin{aligned} ||A||_1^{x,\gamma} &= \sup_{||\varepsilon||_2\leq\gamma} \frac{||A(x+\varepsilon)-A(x)||_1}{||\varepsilon||_2}, \\ ||A||_\infty^{x,\gamma} &= \sup_{||\varepsilon||_2\leq\gamma} \frac{||A(x+\varepsilon)-A(x)||_\infty}{||\varepsilon||_2}. \end{aligned} \tag{10}$$

The first of these compares the size of the average change (*mean absolute deviation*) of the yield curve to the size of the change in the input, while the second ones does the same with the maximum change (*maximal absolute deviation*) in the output. As the notation suggests—due to the nonlinearity of the mapping A—these quantities depend on the original input x and the maximal allowed size of the perturbation ε. We set γ to one half of a basis point, as yields are quoted on a one basis point step grid.

Table 5 gives the condition numbers resulting from random perturbations of all maturities. We calculated the supremum over ten random perturbations for each day in the sample period, and report the percentiles

Table 5

Condition numbers with respect to random perturbations

	SV	SV_FB	SPL	SPL_FB	SPLV	SPLV_FB
	Mean absolute deviation					
Median	0.51	0.67	0.72	0.73	1.09	1.66
90%	0.71	0.92	0.87	0.86	7.83	8.82
95%	0.77	1.11	0.92	0.91	12.00	52.59
97.5%	0.82	2.28	0.99	0.95	60.49	78.24
Maximum	1.25	3.85	1.08	1.02	232.96	312.68
	Maximal absolute deviation					
Median	1.19	1.63	3.61	2.89	7.94	4.87
90%	2.06	2.80	4.48	3.55	61.67	164.18
95%	2.23	3.60	4.77	3.74	278.26	935.74
97.5%	2.37	16.07	4.99	3.89	982.36	2321.26
Maximum	10.72	35.51	5.46	4.40	3851.49	24120.95

of the resulting condition numbers. The same perturbations were used for each estimation method to ensure that the results are comparable. We set $h = 0$ and $H = 20$ in (9), i.e., we measured the change in the yield curves over the whole maturity range.

To interpret the numbers, note that, e.g., the number in the third column of the third row indicates that, when using the SPL method, the average change in the estimated yield curve was less than $0.92 \times 0.5 = 0.46$ basis points on 95% of all the days in the sample period and for all the perturbations examined. On the other hand, the ninth row in the second column shows that when we estimated the yield curves by the SV_FB method, on 2.5% of the days there was a perturbation (less than one half of a basis point in size) that induced an eight (=16.07×0.5) basis point change in the estimated curve at some maturity.

It is apparent that the unsmoothed spline methods give unacceptable results; for example, the seventh row of the fifth column shows that for the SPLV method a half basis point perturbation frequently (once in two weeks on average) causes a thirty basis point shift in the estimated curve at some maturities. This is another indication of the overfitting problem noted earlier.

As for the other four models, the upper rows of Table 5 show that on the average all of them are stable. However, in the worst cases (last row) the Svensson models may produce unacceptable results, while the smoothing spline models retain their stability.

Perturbation of long rates

A more specific exercise was carried out to test the stability of the short end of the estimated curves to perturbations of the long input yields. The perturbations were deterministic: on each day and for each method, one half of a basis point was added to all swap yields—one maturity at a time—and the result with the maximal response was selected. The response was measured on the short end of the curves, i.e., we set $h = 0$ and $H = 1$ in (9).

Table 6 shows a rather different picture that Table 5: it is now the Svensson models that have the worst performance, albeit only in the worst cases. Nevertheless, the possibility of an eight basis point shift in the short end as a result of a half basis point perturbation of a single long swap rate is clearly undesirable. This is a further proof of the inability of the Svensson functional to separate movements at different segments of the maturity range that have been mentioned before.

Table 6

Condition numbers with respect to perturbations of long rates

	SV	SV_FB	SPL	SPL_FB	SPLV	SPLV_FB
	Mean absolute deviation					
Median	0.01	0.02	0.00	0.00	0.01	0.01
90%	0.01	0.10	0.00	0.00	0.04	0.02
95%	0.02	0.12	0.00	0.00	0.04	0.03
97.5%	0.10	0.13	0.00	0.00	0.05	0.03
Maximum	0.26	0.21	0.00	0.00	0.06	0.07
	Maximal absolute deviation					
Median	0.25	0.94	0.24	0.25	0.48	0.55
90%	0.35	6.98	0.24	0.25	1.34	1.12
95%	0.67	8.98	0.24	0.25	1.62	1.45
97.5%	3.83	9.50	0.24	0.25	2.07	1.94
Maximum	11.07	16.84	0.24	0.25	2.39	3.51

Finally, while all four spline models proved to be stable in this test, the almost tenfold relative advantage of the smoothing splines is certainly worthy of note.

SUMMARY OF MODEL PROPERTIES

The results of the previous sections are summarized in Table 7. In the table a −1 indicates that the method was unacceptable by the corresponding criteria, a 0 is given to acceptable but outperformed methods, and 1 was given to the preferred methods.

Table 7

Properties of the estimation methods

	SV	SV_FB	SPL	SPL_FB	SPLV	SPLV_FB
Residuals	1	0	1	0	1	0
Out-of-sample errors	1	0	1	0	−1	1
Forecast errors	1	1	1	1	1	1
Short rates	0	0	1	1	1	1
Stability	0	0	1	1	−1	−1

Only the unsmoothed spline methods ever receive a −1, and this happens with those criteria broadly pertaining to overfitting. Although the use of Fama-Bliss yields does solve the problem of volatile long rates, the stability of the unsmoothed splines remains very poor.

The Svensson functional form was not flexible enough to capture the sometimes heavy curvature at the short end of the curves, while the spline models performed well in this respect, even with relatively few knot points. Lack of flexibility also meant that the separation of the information at the long and the short end of the maturity spectrum was not adequate.

The two smoothing spline methods provided acceptable results according to every criteria, but the smaller out-of-sample errors and better fit seems to favor the smoothing spline model estimated by minimizing pricing errors.

5 Summary

In this paper we gave a brief overview of the most commonly used statistical yield curve estimation methods, presented some key properties of our data, and analyzed the estimation results.

We used interbank rates, forward rate agreement quotes and interest rate swap yields to estimate yield curves. As a preliminary exercise, we checked whether the data coming from different markets are consistent. We found that FRA rates provide unbiased forecasts of future BUBOR rates, and the discrepancy between the BUBOR and FRA rates does not preclude their joint use in the estimation.

We estimated yield curves using three functional forms: Svensson, smoothing splines and unsmoothed splines with variable knots, and two objective functions: fitting to prices and fitting to Fama-Bliss yields. The results were evaluated with respect to the properties of the residuals and out-of-sample errors, unbiasedness of forecasts, and the stability of the estimated curves to perturbations of the input data. We also briefly examined the plausibility of the results.

Both the residuals and the out-of-sample errors showed that the price-fitting methods were better able to capture the data, especially in the long run. The Svensson models proved to be inferior tho the spline models in fitting the short end of the maturity range. A comparison of the residuals and the out-of-sample errors showed that overfitting was a problem only in case of the variable knot spline model.

This finding was corroborated by the results of the stability tests; these tests also emphasized the inflexibility of the Svensson functional form from another point of view: small perturbations of the long input rates often lead to unreasonable variation in the short end of the estimated curves.

All in all, the smoothing spline model proved to be the most suitable method for estimating yield curves on a daily basis.

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Appendices

A Technical details of estimation

OBJECTIVE FUNCTION

To calculate the cashflow matrix C of the notional bonds behind the swap contracts, we have to know the dates when the exchange of interest rates takes place. In case of the one year swap, there are four quarterly exchanges, while for the longer maturity contract exchanges happen semi-annually. Since we only had data for the new issues, the vector of cashflow dates was the same on all days:

$$h = (0.25, 0.5, 0.75, 1, 1.5, \ldots, 19.5, 20).$$

Quoting swap contract in par yields means that the price of the notional bonds is 1. When the BUBOR and FRA yields are also incorporated, the objective function for the price fitting method becomes the following modification of (6):

$$\begin{aligned}\rho(\pi) = (\mathbf{1} - C\,\delta(\pi,h))'(\mathbf{1} - C\,\delta(\pi,h)) + \sum_t \left(B(t) - 1/t \int_0^t f(\pi,s)ds \right)^2 + \\ + \sum_t \left(F(t) - 4 \int_t^{t+3/12} f(\pi,s)ds \right)^2,\end{aligned} \tag{11}$$

where $B(t)$ and $F(t)$ are the BUBOR and FRA yields of horizon t, and $\mathbf{1}$ is a vector of 1-s.

To calculate the Fama-Bliss yields, BUBOR contracts were treated as zero coupon bonds, since from each such contract we can calculate the value of the discount function at one date t. Therefore the vector of Fama-Bliss yields $\phi(h)$ already contains the information in the BUBOR rates, where h is now the union of the h above and the maturities of the BUBOR rates. The objective function with the FRA yields added will then be

$$\rho(\pi) = (\phi(h) - f(\pi,h))'(\phi(h) - f(\pi,h)) + \sum_t \left(F(t) - 4 \int_t^{t+3/12} f(\pi,s)ds \right)^2. \tag{12}$$

The fact that h contains every cashflow date of the swaps and not just the maturities means that we have tried to make our estimated curves incorporate the assumption of a piecewise constant forward curve as much as possible. It is very likely that this is the explanation why the the SPLV_FB curves are much less volatile at the long end than the SPLV curves.

INITIAL VALUES

Most of the estimations had to be carried out by using numerical optimization algorithms, since the derivatives of the objective functions were nonlinear functions of the parameters.[9] It is often found that, when using the Svensson functional form, the result of these algorithms highly depends on the initial values, see Anderson & Sleath (2001) and Gyomai & Varsányi (2002).

One way of controlling this problem is to run the algorithm with several starting values and select the best fitting result. We did this with starting values that were all possible combinations of the numbers in Table 8.

[9] The only exception was the SPLV_FB method, which was estimated by OLS.

Table 8

Initial values of the SV and SV_FB models (729 combinations)

β_0	0	0.05	0.1
β_1	−0.075	0	0.075
β_2	−0.75	0	0.75
β_3	−0.75	0	0.75
τ_1	1	2	4
τ_2	1	2	4

Table 9

Differences of the forward curves

Difference (basis points)	Frequency (%)	
	SV	SV_FB
$[0,1)$	63.97	57.77
$[1,2)$	12.67	12.78
$[2,5)$	15.92	18.34
$[5,10)$	5.78	6.95
$[10,25)$	1.60	3.10
$[25,\infty)$	0.06	1.05

We found that the estimated *parameters* were indeed very sensitive to the starting values. However, when the resulting *forward curves* were compared, the variation proved to be much less significant. More precisely, we estimated the parameters on each day in the sample period with the starting values described above, calculated the absolute deviation of the forward curves on a two-week step grid, and examined the distribution of the pooled results. The percentiles of this distribution are given in Table 9.

We see that in roughly 95% of the cases the absolute difference of the SV curves is less than five basis points, and in almost 65% of the cases it is even below one basis point. Differences larger than ten basis points occur in less then 2% of the cases. The situation is only slightly worse for the SV_FB method. We therefore decided to use the "psychologically simplest" (0, 0, 0, 0, 1, 1) vector as starting value.

CHOICE OF KNOT POINTS

The estimation of spline models is numerically stable, the question here is how to chose the knot points. McCulloch (1971) suggests to use $\sqrt{N}$ knots point placed such that there are $\sqrt{N}$ maturities between successive knots, while Fisher *et al.* (1995) and Anderson & Sleath (2001) place a knot at the maturity of every third bond. We opted for the second approach; however, as this usually results in many knots, a roughness penalty function must be used to control for extra volatility.

After experimenting with a constant penalty λ as in Fisher *et al.* (1995), we found that the resulting forward curves were either "over-smoothed" at the short end or "under-smoothed" at the long end, so we turned to variable rate penalty functions. Among the various smoothing functions found in the literature (Anderson & Sleath (2001), Waggoner (1997), Gyomai & Varsányi (2002)), we decided to use the one in Anderson & Sleath (2001):

$$\lambda(t) = e^{L-(L-S)e^{-t/\mu}}. \tag{13}$$

Figure 8

Penalty function ($L = 2, S = -10, \mu = 2$)

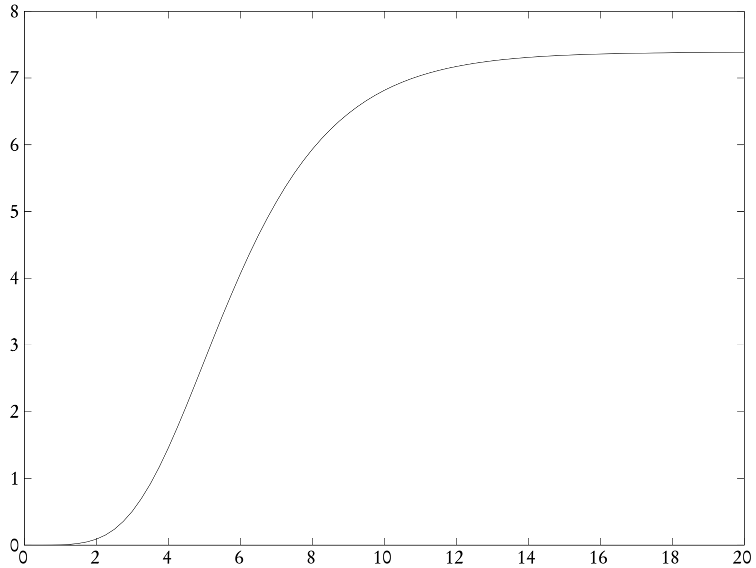


Table 10

Values of the knot points in the SPLV and SPL_FB methods (216 combinations)

x_0	0					
x_2	0.25	0.5	0.75	1	1.25	1.5
x_3	2	2.5	3	3.5	4	4.5
x_4	5	6	7	8	9	10
x_5	20					

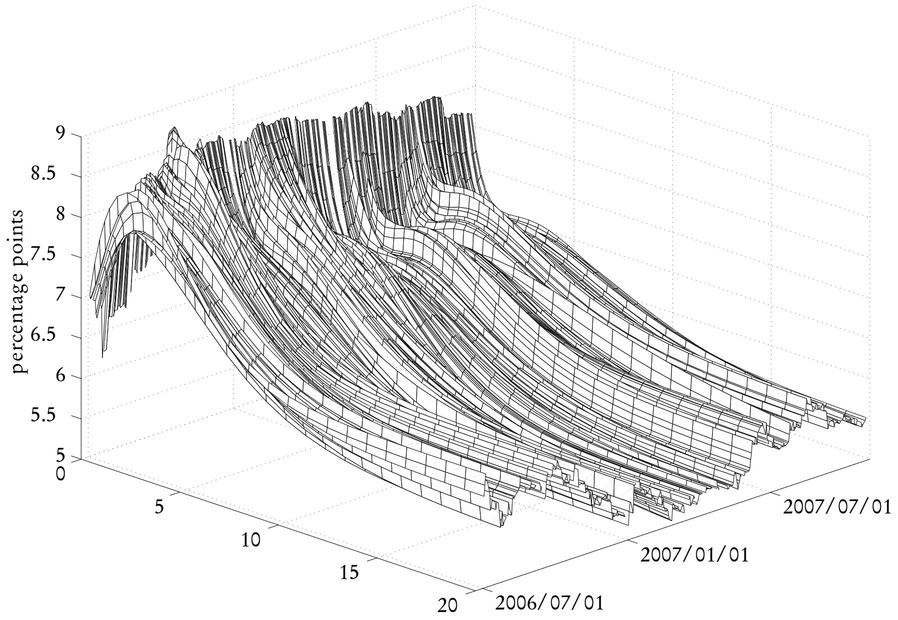
The three parameters could have been ”estimated” on each day by minimizing the out-of-sample errors as in Anderson & Sleath (2001), so that the smoothing function is adapted to the data. However, when we did this, we found that the results with the optimal parameters were hardly different from those obtained by setting $L = 2$, $S = -10$, $\mu = 2$—which were the starting values in the above optimization algorithm—, therefore we used these fixed values. The shape of the corresponding penalty function is shown in Figure 8.

Nevertheless, the choice of the knot points and the penalty function remains ad hoc, therefore we decided to experiment with methods where the placement of the knots is adapted to the data. One such method is presented in Fernandez-Rodriguez (2006), who uses a genetic optimization algorithm to overcome the problem of numerical complexity when the knots are not fixed. Instead of pursuing this line of thought, we decided to follow the much simpler method of taking a fixed grid of knots, estimating the model with these knots and choosing the best fitting result. The grid consisted of the possible combinations of the values in Table 10.

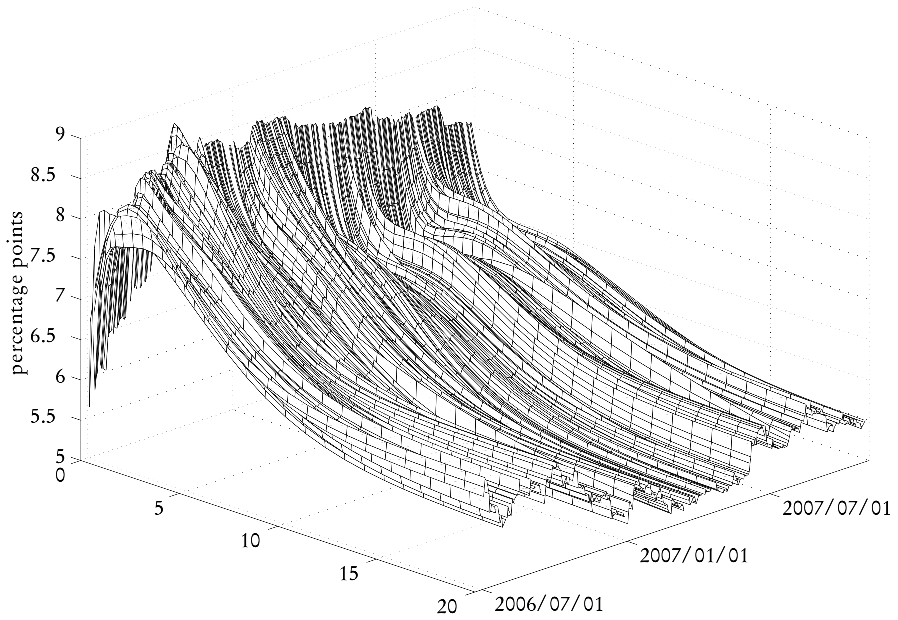
B Figures and tables

ESTIMATED FORWARD CURVES

Figure 9
Forward curves I

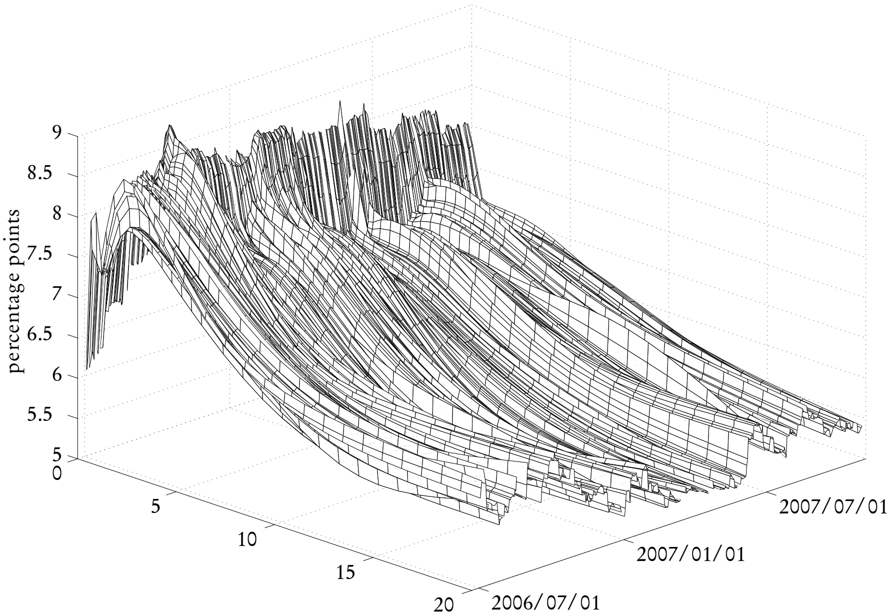


(a) SV

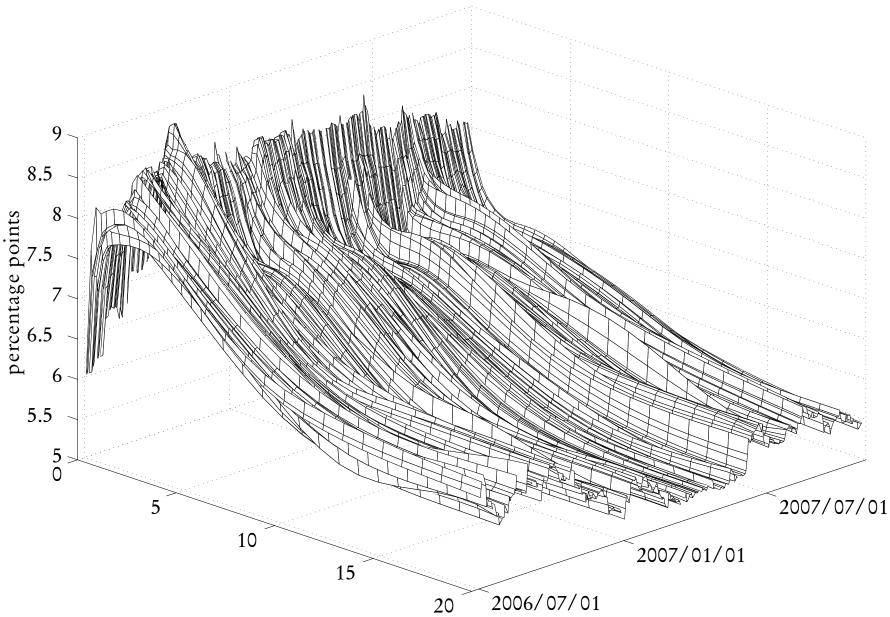


(b) SV_FB

Figure 10
Forward curves II

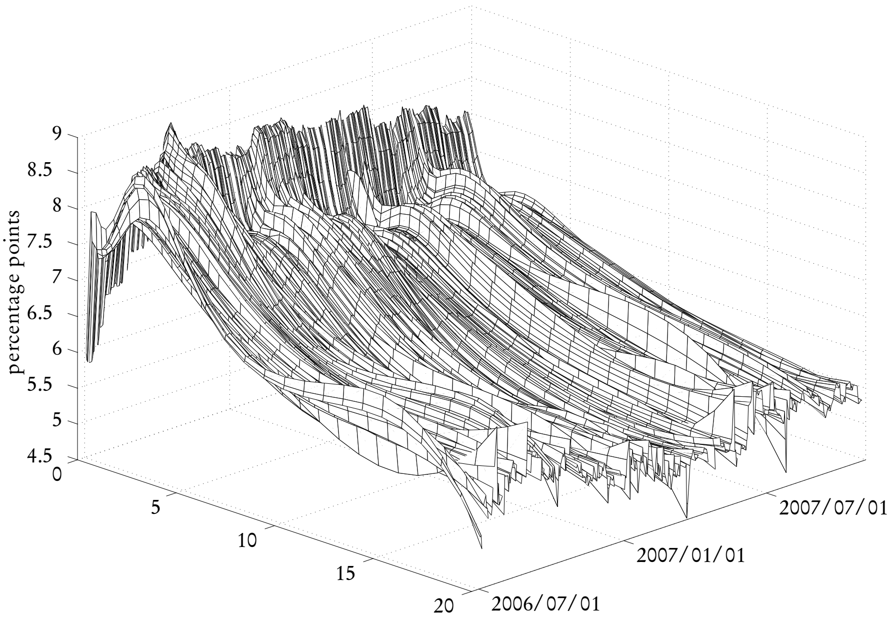


(a) SPL

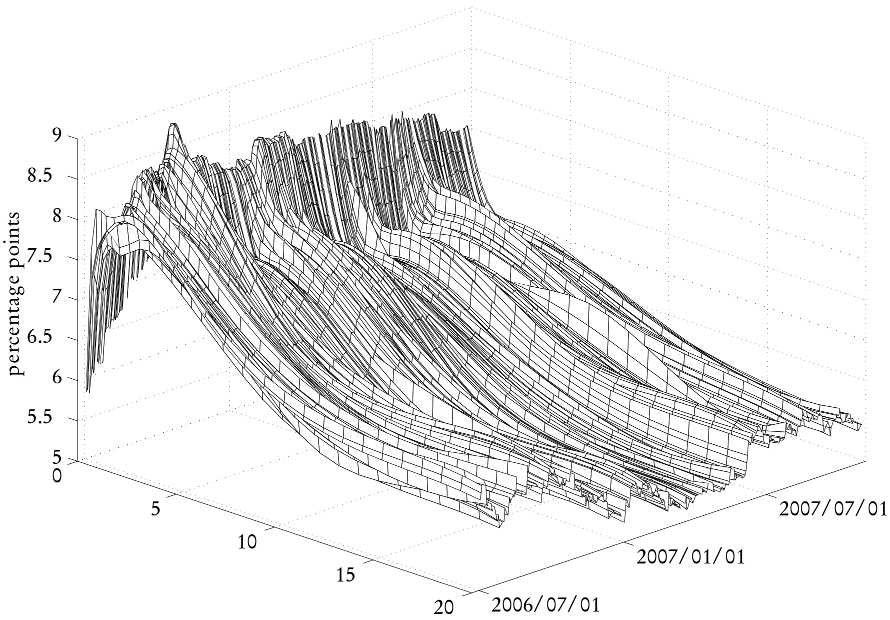


(b) SPL_FB

Figure 11
Forward curves III



(a) SPLV



(b) SPLV_FB

RESIDUALS

Table 11

Swap residuals (basis points)

Maturity	SV	SV_FB	SPL	SPL_FB	SPLV	SPLV_FB
1	20.25	16.50	19.79	16.18	19.08	15.98
2	15.30	14.70	13.58	13.21	19.61	15.28
3	11.47	19.81	7.53	14.70	10.75	15.60
4	6.15	21.07	3.04	14.01	2.40	13.62
5	3.41	19.89	1.98	13.06	5.35	12.28
6	4.66	17.60	2.37	12.78	4.82	11.81
7	6.21	17.20	3.96	13.72	4.18	12.75
8	5.44	18.14	3.74	14.54	3.44	13.70
9	4.62	20.72	3.93	16.01	4.31	15.36
10	6.03	24.29	5.43	17.82	5.62	17.52
12	7.97	29.38	6.61	19.53	5.92	18.45
15	5.59	30.65	2.99	18.71	3.08	16.94
20	4.14	28.68	1.34	19.35	0.45	18.45

Table 12

BUBOR residuals (basis points)

Maturity	SV	SV_FB	SPL	SPL_FB	SPLV	SPLV_FB
2w	20.86	20.51	4.28	4.20	3.10	7.44
1m	17.60	17.24	5.27	4.75	4.37	5.99
2m	12.16	12.80	6.10	4.91	7.25	5.37
3m	7.61	10.26	5.74	6.40	7.08	6.92
4m	3.96	7.47	4.47	6.11	5.12	6.57
5m	3.96	6.69	2.62	7.08	2.58	7.30
6m	7.27	7.88	3.14	8.57	3.22	8.65
7m	10.11	8.87	5.90	9.08	5.87	9.05
8m	12.48	10.15	9.09	9.95	8.55	9.80
9m	15.05	11.93	12.76	11.45	11.70	11.21
10m	16.92	13.14	15.61	12.58	14.31	12.27
11m	18.93	14.70	18.26	14.16	17.10	13.86
12m	20.88	16.26	20.57	15.85	19.81	15.63

Table 13

FRA residuals (basis points)

Maturity	SV	SV_FB	SPL	SPL_FB	SPLV	SPLV_FB
1 × 4	7.02	5.96	10.95	4.51	12.16	5.52
2 × 5	7.29	6.14	12.43	6.79	11.79	7.50
3 × 6	8.77	9.45	12.55	10.98	10.84	11.56
4 × 7	9.19	12.71	11.19	15.04	10.37	15.62
5 × 8	9.67	15.49	8.32	18.28	9.96	19.01
6 × 9	10.63	18.15	6.28	20.84	9.90	21.81
7 × 10	11.71	20.22	5.06	22.30	9.37	23.38
8 × 11	13.47	22.12	5.75	23.15	9.01	23.95
9 × 12	15.46	23.76	8.65	23.59	8.90	23.63
10 × 13	16.74	24.74	11.57	23.35	8.40	22.33
11 × 14	17.48	25.21	13.86	22.56	8.05	20.39
12 × 15	18.29	25.68	15.88	21.77	8.75	18.59

OUT-OF-SAMPLE ERRORS

Table 14

Swap out-of-sample errors (basis points)

Maturity	SV	SV_FB	SPL	SPL_FB	SPLV	SPLV_FB
1	20.92	16.50	20.64	16.18	19.86	15.98
2	17.16	14.72	15.97	13.72	23.19	16.45
3	14.33	20.07	10.38	15.74	13.54	17.24
4	7.91	21.80	4.11	14.71	3.68	14.66
5	4.38	19.85	2.72	13.40	7.70	12.72
6	5.87	17.64	3.12	12.87	6.30	11.88
7	7.63	17.09	5.11	13.86	5.43	12.94
8	6.57	23.02	4.85	14.48	4.67	13.66
9	5.74	20.69	5.14	15.99	5.85	15.29
10	7.77	24.42	7.46	18.17	8.19	17.77
12	11.28	29.18	10.28	20.11	10.47	19.14
15	8.32	30.58	8.56	18.22	30.52	16.73
20	–	–	–	–	–	–

Table 15

BUBOR out-of-sample errors (basis points)

Maturity	SV	SV_FB	SPL	SPL_FB	SPLV	SPLV_FB
2w	–	–	–	–	–	–
1m	21.03	20.13	7.56	6.21	6.27	8.09
2m	13.55	13.66	7.71	5.21	8.50	5.92
3m	8.36	10.46	6.81	6.27	8.05	6.84
4m	4.27	7.37	5.11	6.48	5.68	7.03
5m	4.24	6.99	2.89	7.68	2.82	7.89
6m	7.70	7.87	3.39	8.63	3.47	8.68
7m	10.60	9.47	6.30	9.59	6.26	9.53
8m	13.05	10.81	9.63	10.43	9.04	10.41
9m	15.65	11.97	13.45	11.47	12.31	11.23
10m	17.56	13.81	16.39	13.32	14.99	12.95
11m	19.68	15.53	19.13	15.07	17.87	14.68
12m	21.68	17.56	21.50	17.22	20.66	16.99

Table 16

FRA out-of-sample errors (basis points)

Maturity	SV	SV_FB	SPL	SPL_FB	SPLV	SPLV_FB
1 × 4	7.84	6.72	13.64	5.46	14.45	6.21
2 × 5	8.10	6.63	15.22	7.94	13.74	8.30
3 × 6	9.34	10.41	15.19	12.54	12.58	12.73
4 × 7	9.83	13.89	14.06	17.31	11.95	17.11
5 × 8	10.32	16.75	10.07	20.50	11.28	20.82
6 × 9	11.45	19.08	7.36	22.83	11.19	23.72
7 × 10	12.66	21.39	6.02	24.63	10.48	25.25
8 × 11	14.39	23.62	6.84	25.66	10.11	25.84
9 × 12	16.50	25.38	10.05	25.85	10.07	25.69
10 × 13	17.70	26.53	13.04	25.43	9.64	24.79
11 × 14	18.58	26.91	15.48	24.76	9.41	23.49
12 × 15	19.68	27.99	17.89	24.39	10.62	22.50

FORECAST ERRORS

Table 17

Forecast errors of 3M BUBOR rate, mean (basis points)

Horizon	SV	SV_FB	SPL	SPL_FB	SPLV	SPLV_FB
1m	0.18	0.68	−4.63	3.23	−5.63	2.61
2m	6.66	4.86	−2.32	4.48	−1.70	3.35
3m	10.60	6.83	−0.13	3.01	1.78	2.57
4m	6.45	1.35	−2.36	−4.79	−1.14	−4.75
5m	1.64	−5.00	−3.04	−12.07	−4.04	−11.92
6m	−3.13	−10.78	−3.30	−17.97	−6.32	−17.77
7m	−6.08	−15.25	−2.91	−21.25	−6.51	−21.22
8m	−15.27	−25.19	−10.85	−29.13	−12.84	−29.24
9m	−20.50	−31.11	−16.21	−31.98	−15.70	−32.21
10m	−22.11	−33.58	−19.16	−31.78	−16.18	−32.00
11m	−29.28	−41.16	−27.68	−37.24	−23.02	−37.00
12m	−38.69	−50.58	−39.64	−46.07	−34.18	−45.16

Table 18

Forecast errors of 3M BUBOR rate, RMSE (basis points)

Horizon	SV	SV_FB	SPL	SPL_FB	SPLV	SPLV_FB
1m	17.59	19.58	19.27	19.23	19.75	18.88
2m	29.61	28.99	29.11	29.72	29.41	29.35
3m	35.82	32.93	34.00	33.59	34.96	33.72
4m	33.91	31.43	35.14	33.85	35.47	33.81
5m	33.96	32.20	36.73	36.32	36.70	36.26
6m	33.37	34.12	36.44	38.48	35.97	38.58
7m	33.13	35.28	35.70	38.96	34.95	39.09
8m	37.14	42.22	38.84	44.94	38.62	45.41
9m	38.55	45.61	39.63	46.43	39.38	46.94
10m	40.46	48.68	42.24	48.16	41.98	48.74
11m	41.22	50.95	42.37	48.63	40.77	48.69
12m	44.93	56.42	47.01	52.59	44.07	51.75